THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 58, PART 3 — 1 May 1946 — No. 327

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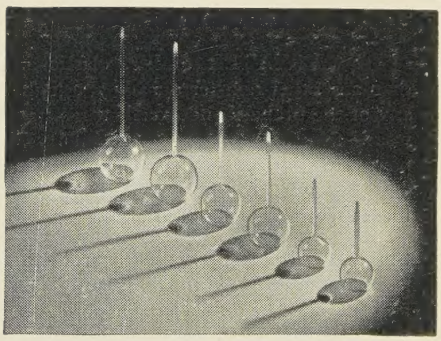

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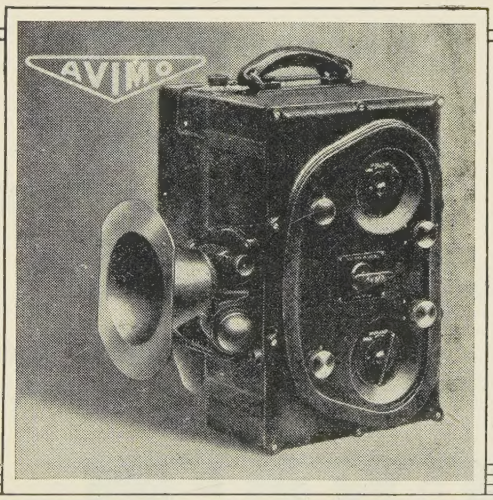

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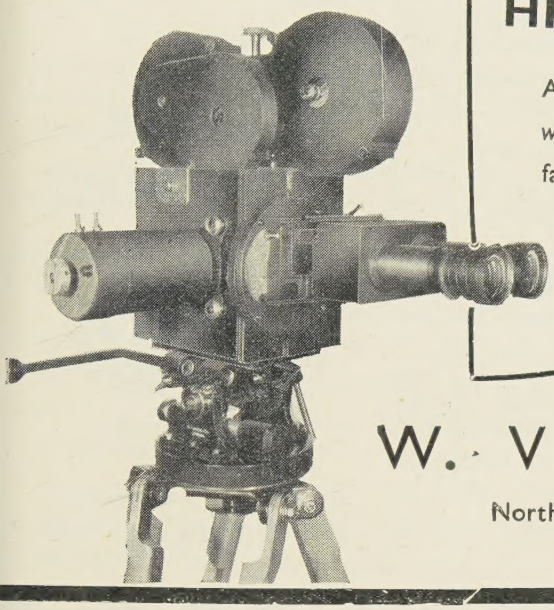

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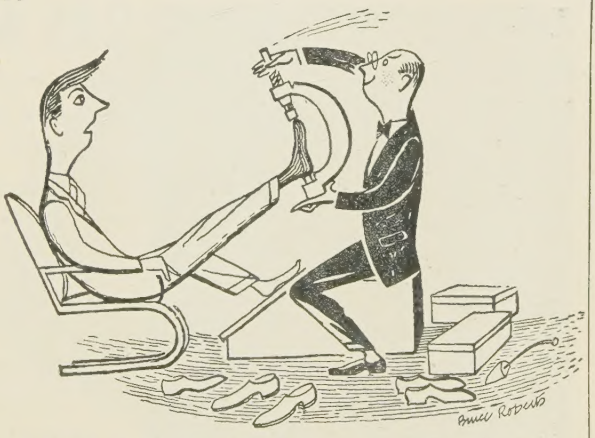

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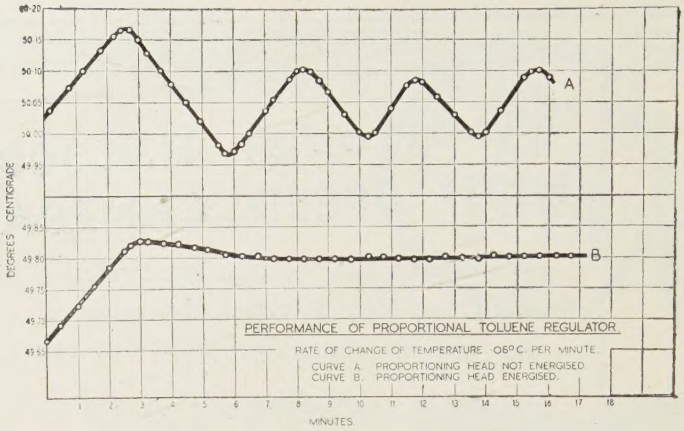

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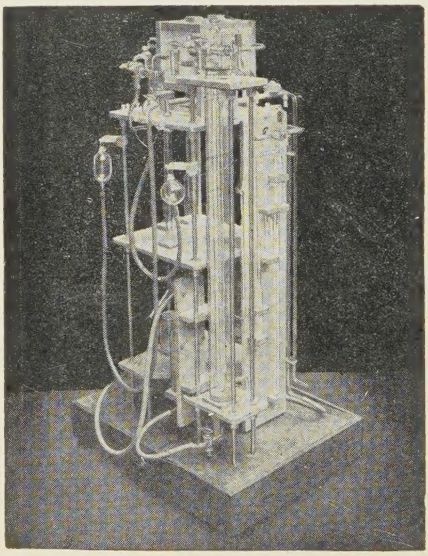

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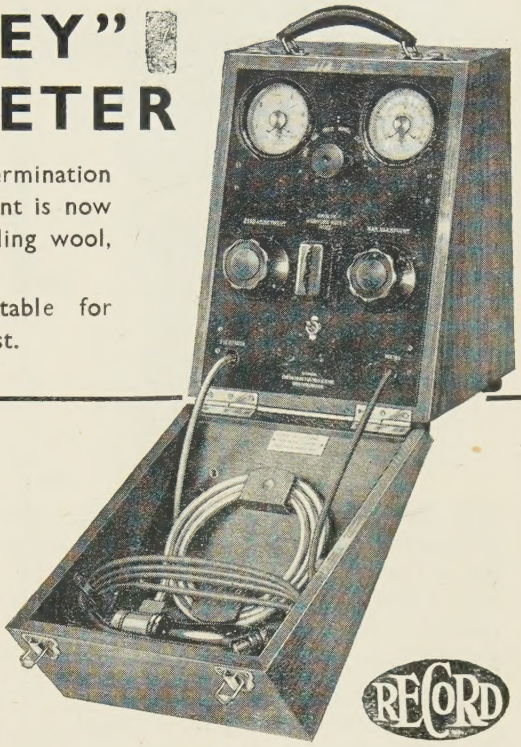

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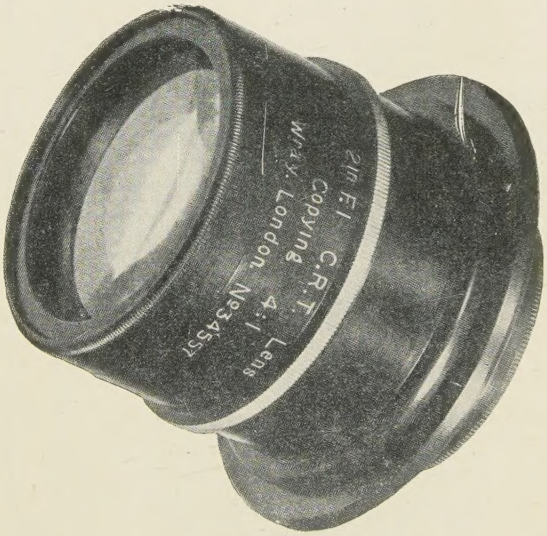

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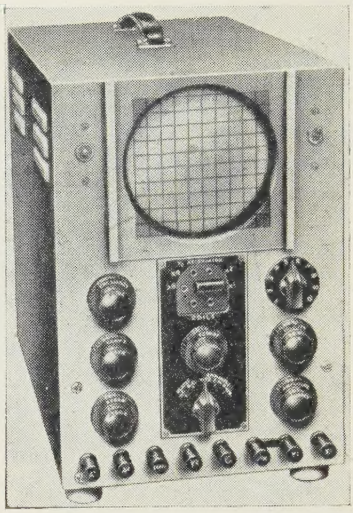

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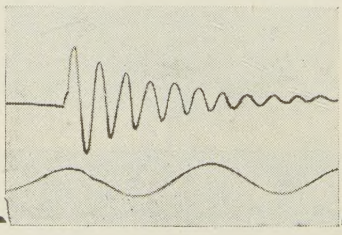

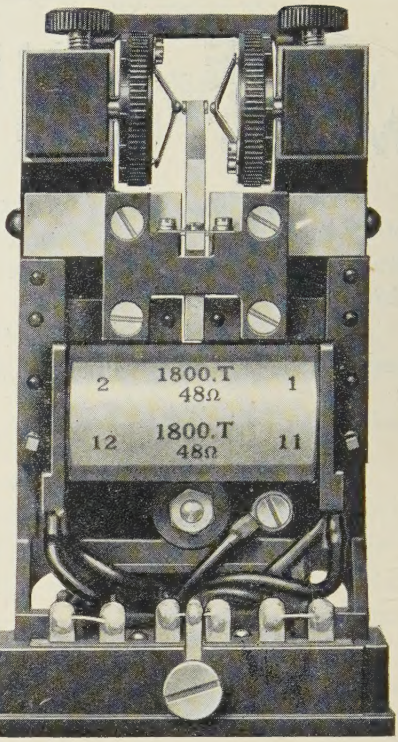

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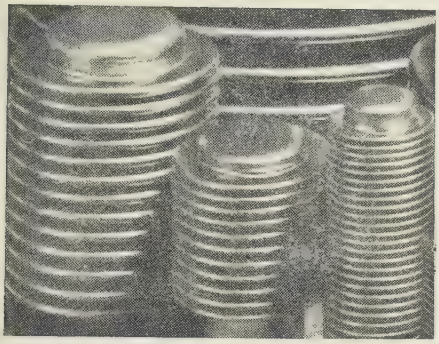

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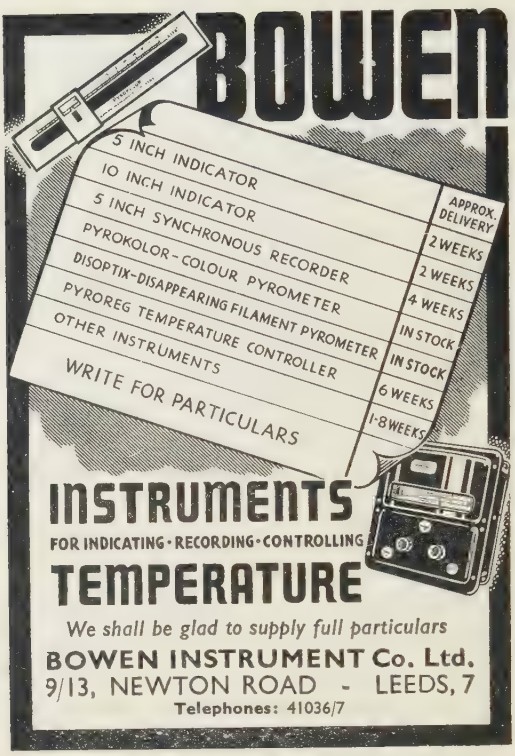

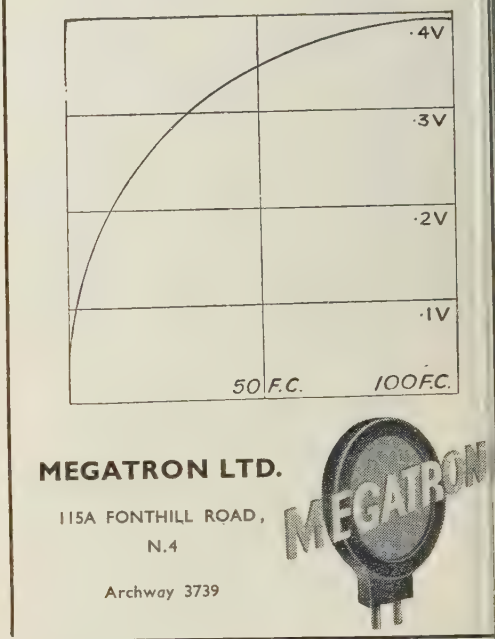

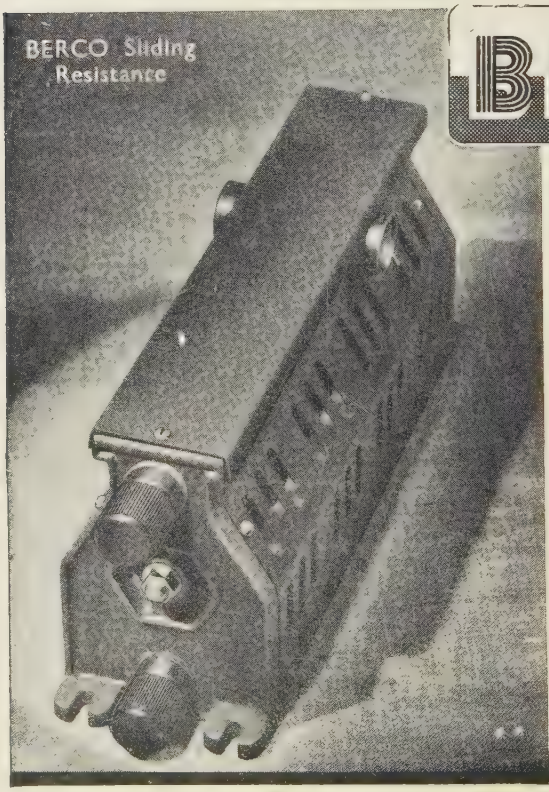

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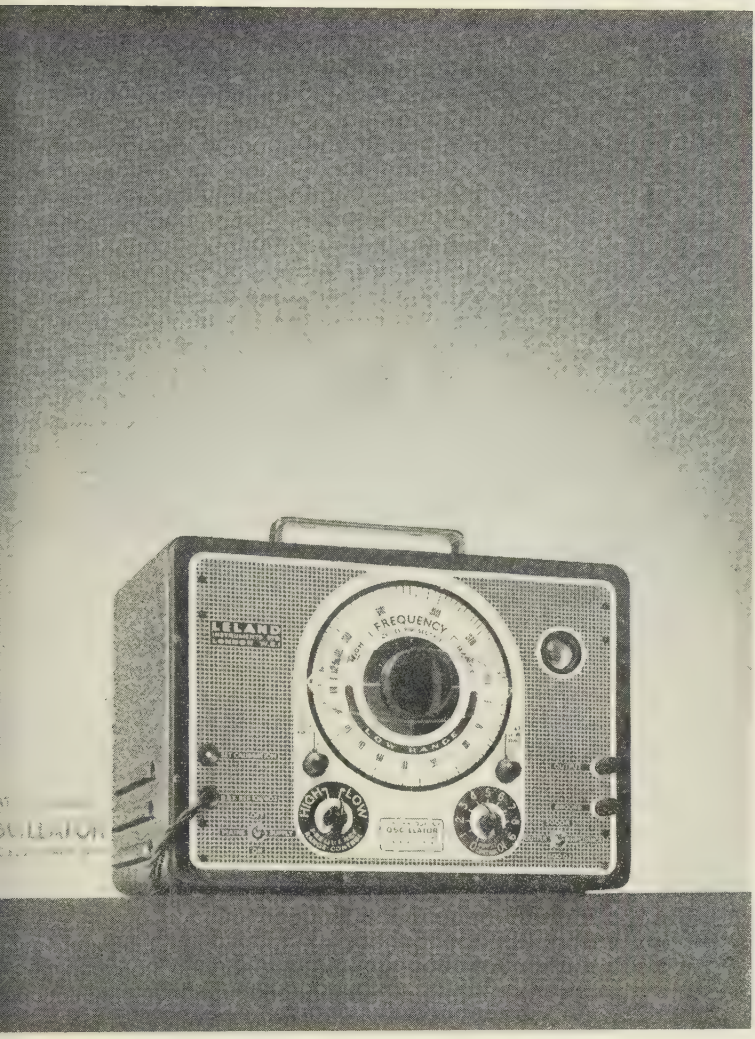

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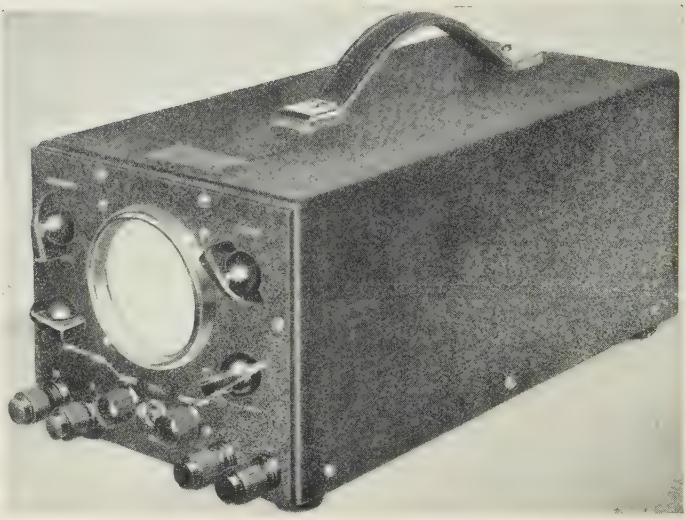

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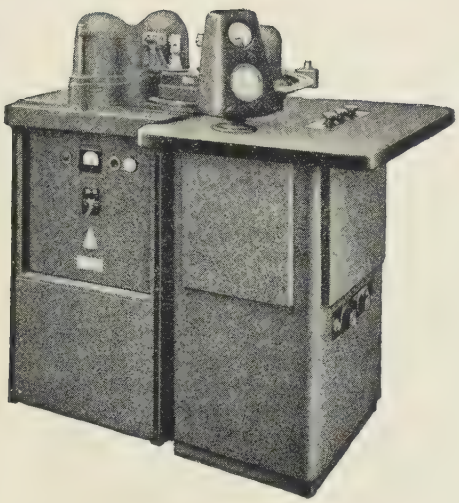

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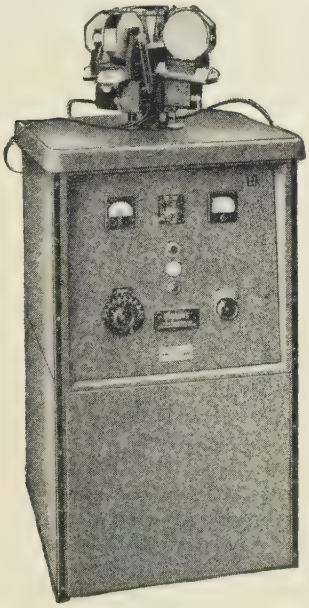

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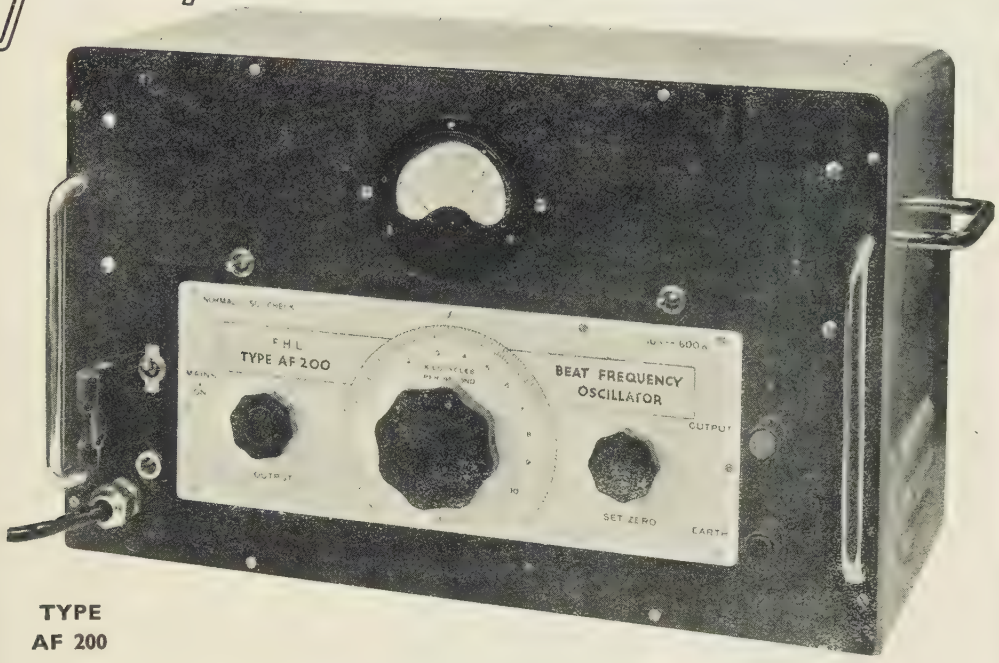

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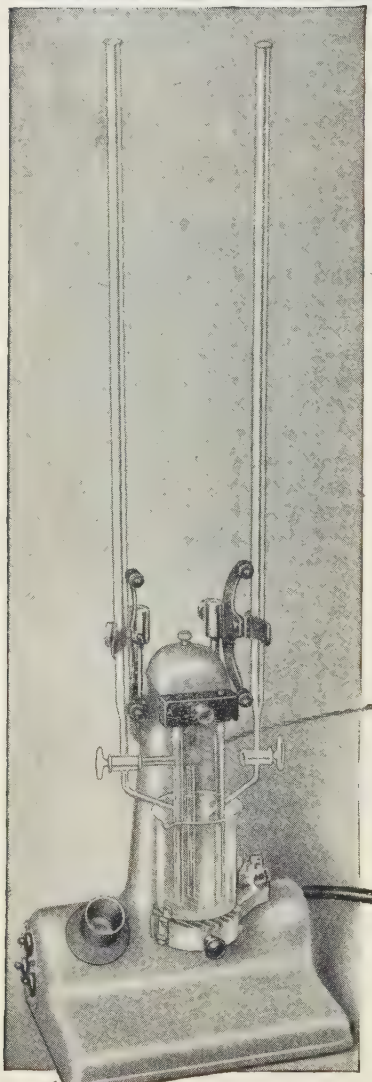

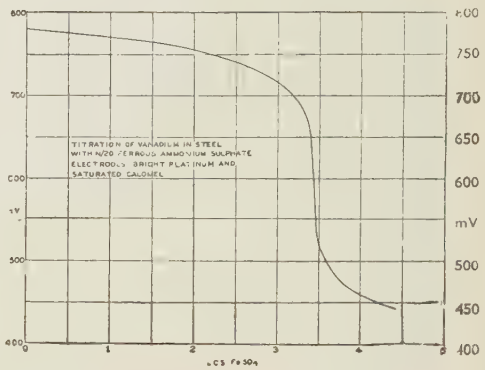

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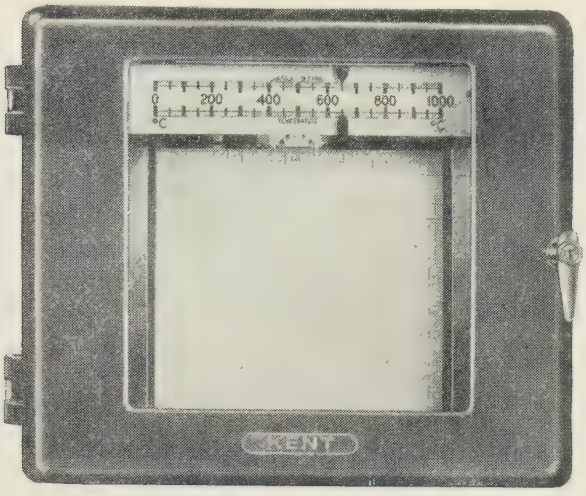

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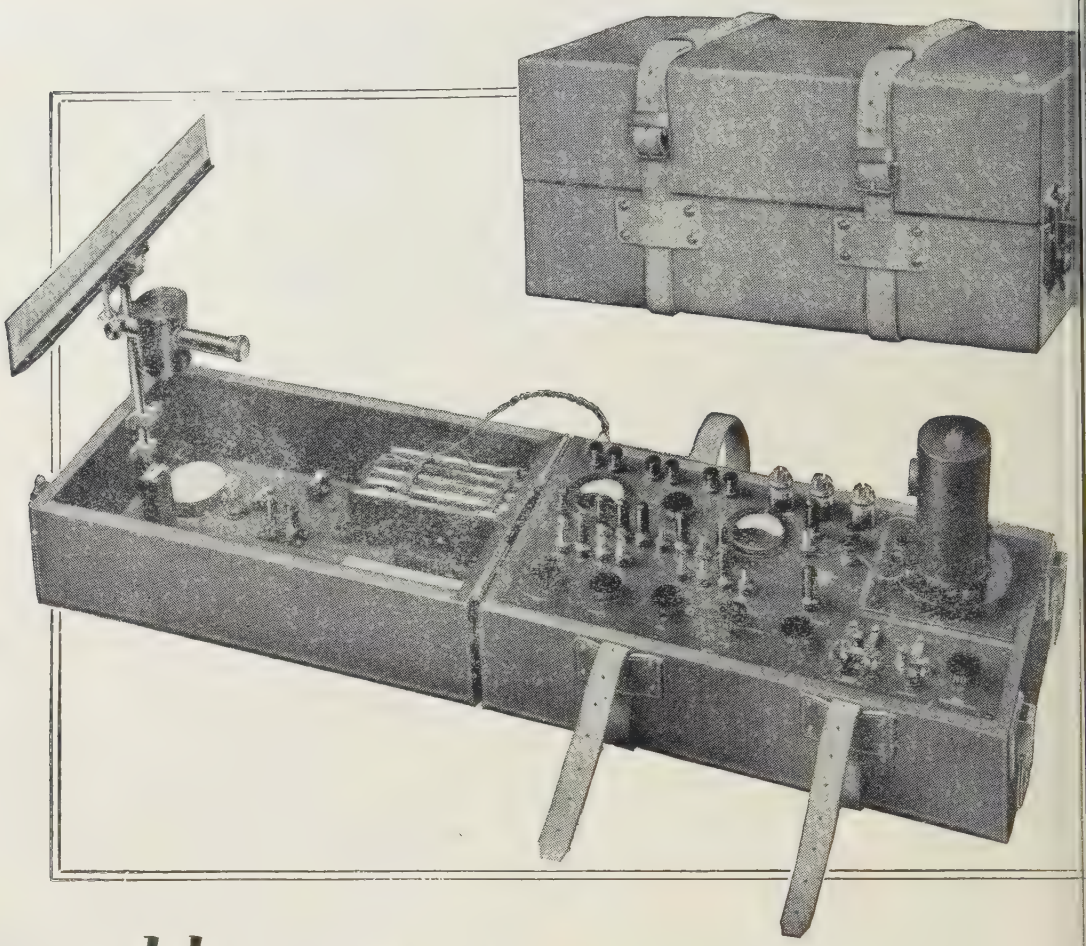

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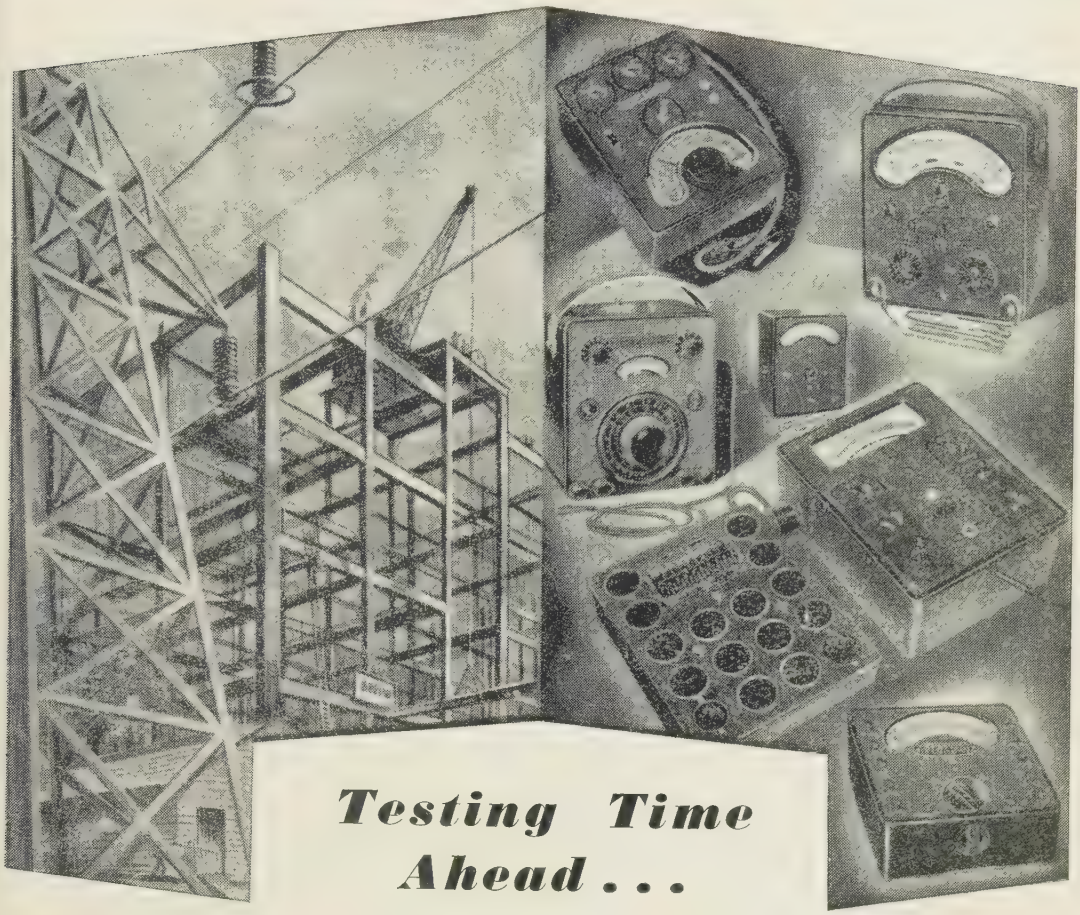

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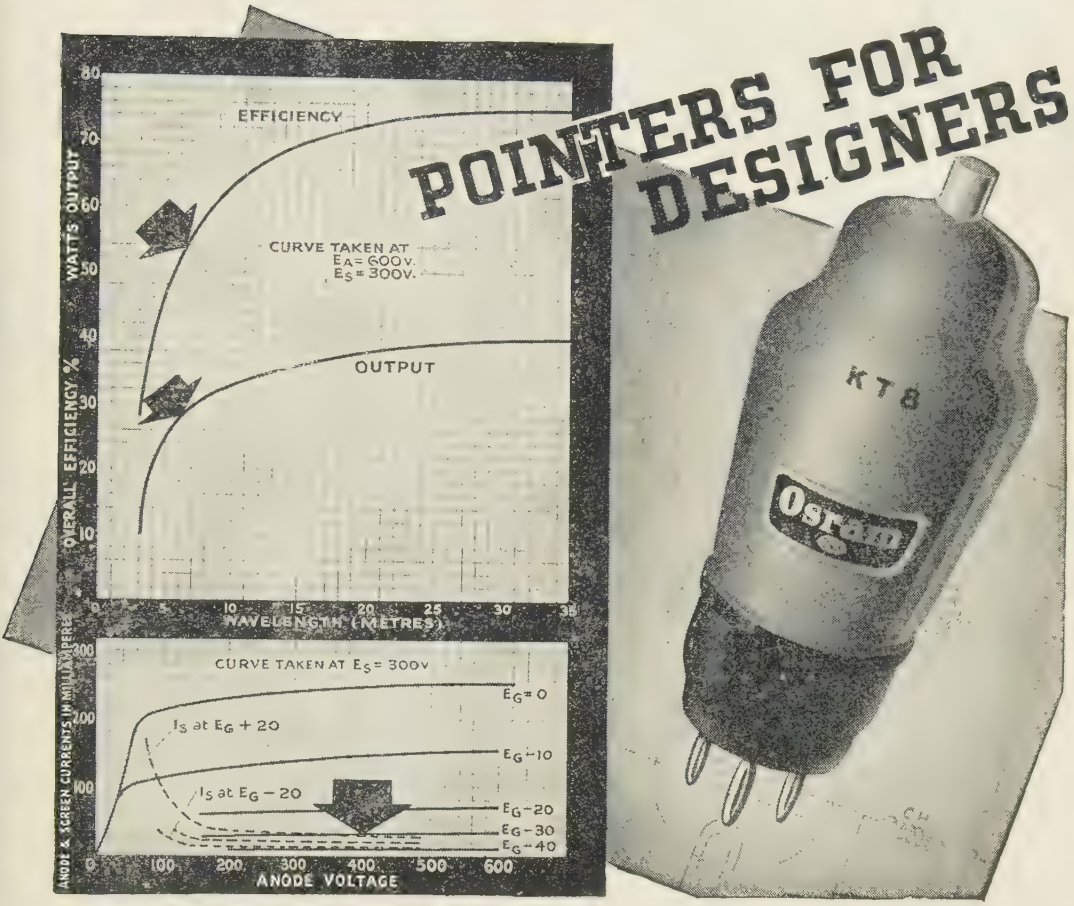

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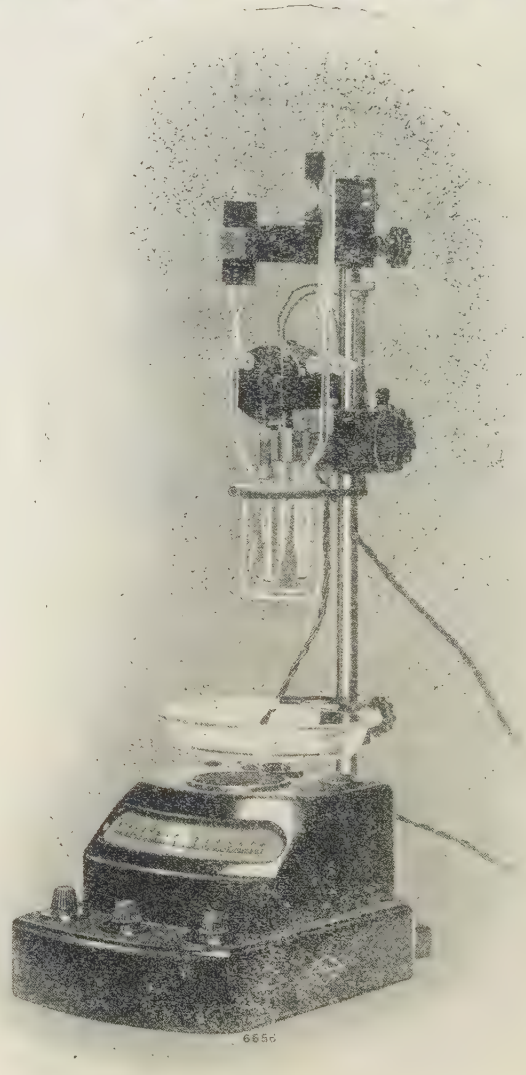

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THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 58, PART 3 1 May 1946 No. 327

GEOMAGNETIC SECULAR VARIATIONS AND SURVEYS

BY J. A. FLEMING,

Director, Department of Terrestrial Magnetism,
Carnegie Institution of Washington

Third Charles Chree Address, delivered 6 December 1945

ABSTRACT. The secular variations of the Earth's field call for frequent determinations of the geomagnetic elements at many selected stations on land and at sea. So far, general world magnetic surveys have of necessity been restricted to the surface of the Earth. Previous surveys, including those of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, have led to great improvement in our knowledge of the distribution of the field, especially during the past few decades. During the war, much attention has been devoted to the study and analysis of data well distributed over the Earth's entire surface at more than 10,000 stations. These have resulted, for the first time, in the preparation of accurate isoporic charts, that is, charts of equal annual rate of change for magnetic declination, inclination, and the horizontal, vertical, eastward, northward and total components of the field for the four epochs of 1912·5, 1922·5, 1932·5, and 1942·5. The motions of the maximum and minimum isoporic foci during these four epochs indicate the complexity of the secular changes and interpretations.

Isomagnetic charts based upon surface observations must always be limited, so far as faithful depiction of the field is concerned, because of the impracticability of obtaining observations at an infinite number of stations and of deductions for both the regular and irregular changes in the field. Progress in instrumentation during the war on the applications of the geomagnetic field of the Earth have produced improvements which make feasible the early realization of magnetic surveys by airplanes at several different levels proposed by Professor Sydney Chapman in the first Charles Chree Address. Some of the potentialities, possibilities, and needs for intense national and international coordination in magnetic surveys by plane are reviewed.

§1. INTRODUCTION

I AM humbly grateful to your Society for making me the third recipient of the Charles Chree Medal and Prize. The award is, I take it, largely a recognition of the geomagnetic researches which my colleagues and I have made at the Department of Terrestrial Magnetism during more than four decades with the generous support of the Carnegie Institution of Washington. The two previous awards were to scientific colleagues and friends (see Chapman, 1941, and Schonland, 1943), whose work has constantly inspired investigators everywhere, and especially those of us in the United States who have been associated with the Department of Terrestrial Magnetism. The award affords further evidence of that scientific co-operation of British and American physicists and geophysicists which has been so vital in forwarding our common war effort, particularly in geomagnetic applications.

Progress in geomagnetism owes much to Charles Chree as Superintendent of the Magnetic and Meteorological Observatory at Kew from 1893 to 1925, and thereafter until his death in 1928. He and his contemporaries—Schmidt, Bauer, Van Bemmelen, and Moos—built well on foundations laid by their eminent predecessors in the task of portraying the complex phenomena of the magnetic and electric fields of the Earth through surveys on land and sea, investigations in the laboratory, and theoretical hypotheses and interpretation. As a colleague and friend, Chree influenced greatly the work of Bauer in 1899 through the expansion of geomagnetic research in, and survey of, the United States Coast and Geodetic Survey, and again in 1904 through the realization of Bauer's plan for a Department of Research in Terrestrial Magnetism submitted to the Trustees of the Carnegie Institution of Washington. His wise counsel carried great weight and played a large part in the subsequent development and growth of the Department. Chree's share in international and national progress in our field is well known to all of you. His commonsense treatment of factual information gathered in all parts of the world—especially in polar regions—was outstanding; it has contributed much to applications of geomagnetic research touching so many lines of human peace-time and war-time endeavor.

The foundation and provision by Miss Jessie Chree of the Charles Chree Medal and Prize through the Physical Society not only serve to honour the memory of Chree's accomplishments but also to stimulate future workers and progress in our fields.

The first Charles Chree Address, in 1941, by Professor Sydney Chapman gave a scholarly general account of the magnetic field of the Earth. I have chosen for my subject *Geomagnetic secular variations and surveys*, and will attempt to review at greater length world magnetic surveys and potentialities so prophetically outlined in Part III of the first Address.

§ 2. GEOMAGNETIC SECULAR VARIATIONS

Several centuries of speculation and research on the old and challenging problem of the origin of the Earth's magnetism and its changes have yielded as yet no adequate explanation. Much has been learned from past attempts at explanations, and we have also added, in the course of time, much new and rich material relating to the description of the phenomenon. This material has illuminated greatly the picture of what we are trying to explain, but understanding of the cause of geomagnetism is emerging relatively slowly.

In spite of vast improvements in our knowledge of the geomagnetic field, that knowledge remains insufficient for various theoretical and practical problems. For example, Schrödinger (1943) has recently attempted to use the data from geomagnetism to test some theories of physical fields which he has developed. For these tests he required information on the magnitude of the external field and of the non-potential field. It was important to know the magnitudes of these quantities to a moderate degree of precision, but those who are working in geomagnetic research recognize that we cannot even be sure, on the basis of existing observations, that there is an external field or a non-potential field.

The magnetic field varies with time (Fleming, 1938a; Barker and McNish, 1938). Its delineation at any instant is hence difficult because it is not practical

to undertake its simultaneous measurement at all points of the Earth's surface. There are some 70 magnetic observatories (figures 1 and 2) where continuous simultaneous measurements of the field are made, but the complexity of the field and its changes with time do not permit satisfactory interpolation of values in intervening regions. The records at observatories must be supplemented by observations at many isolated points. The density of observations, however, per given area of survey is uneven, and great oceanic areas are sparsely represented. Measurements at these stations on land or sea are usually made on one day only, and the resulting data form the basis for isomagnetic charts.

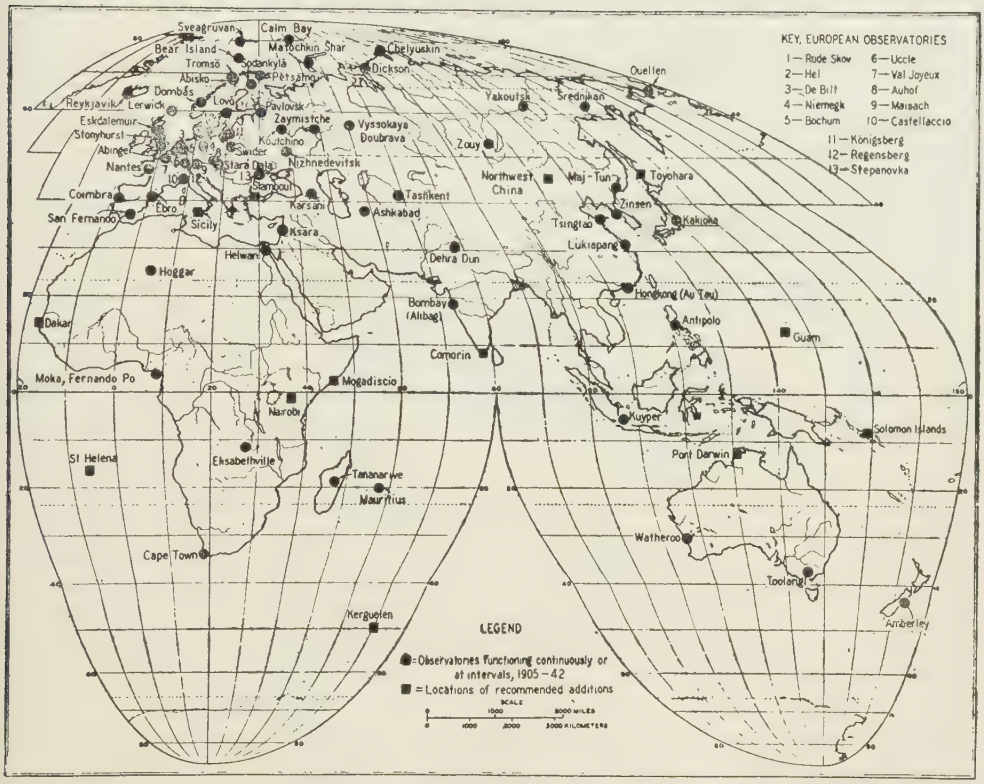

Figure 1. Geomagnetic observations, Eastern Hemisphere, functioning continuously or at intervals, 1905–42, and recommended additional locations.

The world magnetic survey of the Carnegie Institution of Washington from 1905 to the beginning of the war improved knowledge of the general field and its changes, which are slow when compared with the life-span of a human being but are very rapid when compared with astronomical or geological scales of time. Nor are these secular changes small. During the past half-century a gradual change amounting to 10% of the maximum value of the Earth's field has occurred off the Guinea coast. By way of further illustration, consider a few indications based on long series of observations. The actually observed changes in magnetic direction at London and at several other places as determined from earliest known measurements, including those for 1945, are shown in figure 3. The figure shows also

notable changes in the position of the magnetic equator since 1700. The progression in direction of the compass by centuries is non-uniform and notably large. The general aperiodic character of secular change in direction is confirmed by the measured directions of magnetization of particles locked in annual varved deposits in lakes and age-old deposits in oceans.

Radical changes in the estimated position of the lines of zero magnetic declination (agonics) are noted since the year 1500. The line along which the compass

Figure 2. Geomagnetic observations, Western Hemisphere, functioning continuously or at intervals, 1905–42, and recommended additional locations.

points true north (figure 4) has shifted westwards from Suez to Chicago in 445 years, only a short interval in the Earth's history.

Clearly, secular change involves the general magnetic field of the Earth, and any attempt to explain either phenomenon must also account for the other.

There are two hopeful present approaches. One is the derivation of more adequate and complete data on the observed large-scale changes in the Earth's magnetism. The other is the improved description of the permanent field itself.

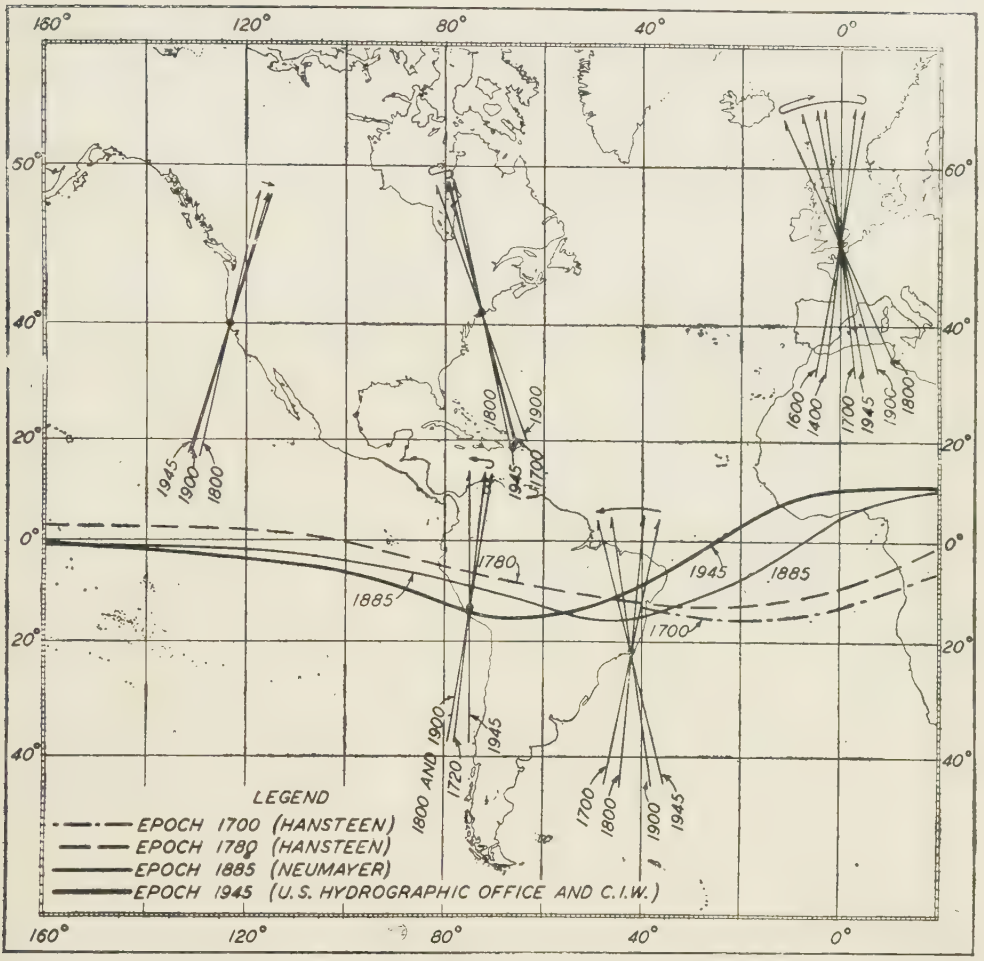

Figure 3. Secular variation of magnetic equator in Western Hemisphere from 1700 to 1945 and of magnetic declination at London, Eastport, San Francisco, Callao, and Rio de Janeiro, from earliest observed values.

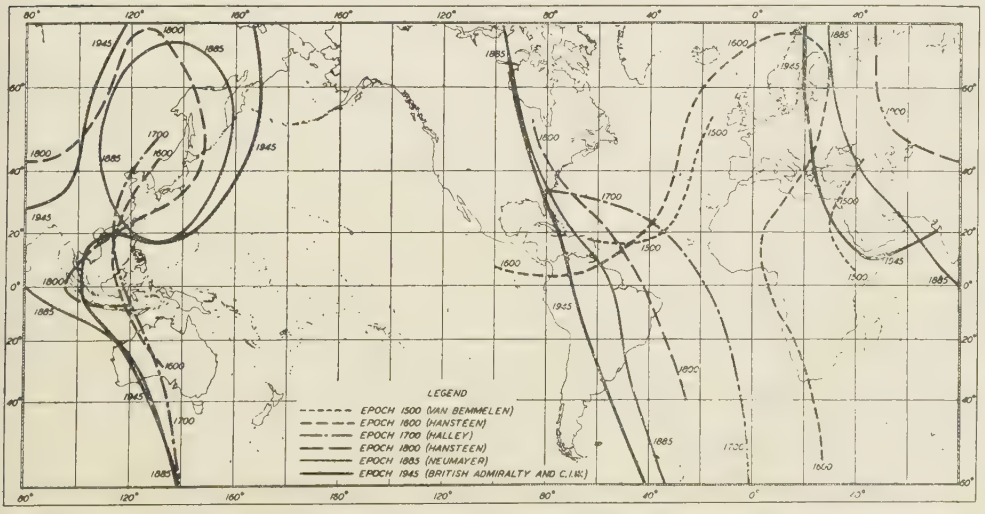

Figure 4. Progressive change (secular variation) of lines of zero magnetic declination (agonics). (The dots indicate the side of the agonic lines on which the declinations were easterly.)

The special type of the research required calls in a unique degree for world-wide coordination of data and experiments. No single, well-planned experiment or observation can supply the solution. Observations must be made in all parts of the world and continued for a long period of time. Techniques for the organization and interpretation of the data must be developed and experimental research must be conducted along lines which will supply information on basic physical problems related to this subject.

In the reduction of magnetic observations to epoch, the aim is to determine at all stations (at epochs not too far removed from those at previous times of observation) the intensity and direction of the Earth's field freed from the effects of small superposed fluctuations in intensity. For practical reasons the problem of describing the Earth's surface field is simplified by mapping only the main or permanent field. The strength of the main field at any time is called its *normal value* at that time.

Although the rough and general description of the geomagnetic field in terms of its normal value as shown on magnetic charts is comparatively easy, the derivation of the normal values themselves is complex and difficult. To obtain these normal values it is first necessary to remove from the individual magnetic observations the contributions of extraneous fluctuations not a part of the secular variation. Often the extraneous fluctuations are large enough to affect seriously the estimates of secular change obtained from two or more observations made in different years at the same place. These fluctuations may thus make difficult or impossible a reliable estimate of the normal values (since the secular change may not be determined accurately either with respect to magnitude or sign) at epochs other than those of observation. When the secular change has been corrected for fluctuations in field, so that it is known with accuracy, the magnetic observations are then readily reduced to the epoch desired for a magnetic chart.

Of fundamental importance in charting the Earth's magnetic field for a given epoch are charts showing rates of annual secular change, that is, isoporic charts. Too little attention has been given by various organizations responsible for the preparation of world isomagnetic maps to the importance of constructing isoporic charts. This importance arises because it provides the only feasible means of enhancing and extending the value measured at a station, say in 1920, to determine the value for the station at a later time, say 1945.

The first comprehensive world isoporic charts were prepared by Fisk (1929 and 1932) and by Fisk and Fleming (1931) at the Department of Terrestrial Magnetism for the epoch 1922·5 (specimen given in figure 5). These give the average annual secular change in the magnetic elements and their components, namely, declination, inclination, and the horizontal, vertical, total, northward, and eastward components of the field.

Fisk's isoporic charts, which have been widely used, were naturally tentative in a number of regions. They did not take into account corrections of the observed data for geomagnetic fluctuations. They are also in several respects mutually inconsistent with the known nature of the geomagnetic field; and the singularities in field near the poles could not be taken into account because of the sparsity of data in high latitudes.

The danger of using these charts today in mapping is that the pattern of the

isopors changes rapidly. For instance, in declination there is scarcely the slightest resemblance between the isopors of 1922 and those of 1942 in many parts of the world. Uncertainties of this kind account for relatively large discrepancies on some charts to actually observed values. In other words, it is necessary to know

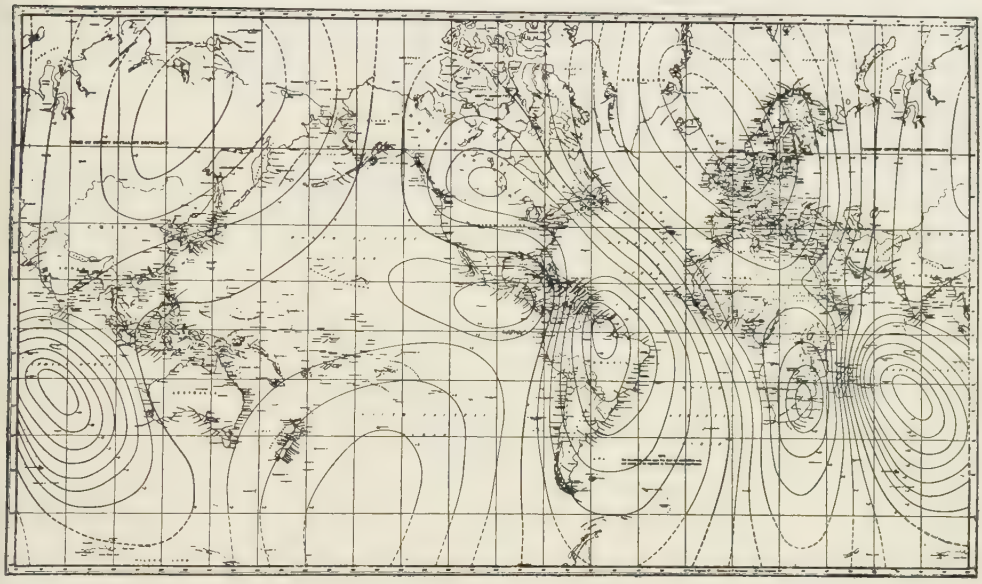

Figure 5. Isoporic chart for magnetic declination in minutes per year, approximate epoch 1922. Prepared at the Department of Terrestrial Magnetism, Carnegie Institution of Washington, by H. W. Fisk, 1930 (slightly revised, October 1942).

not only the pattern of the isopors at a given time but also the trend of change in pattern. This trend can only be derived by mapping the isopors for several different epochs. This in turn requires that there be maintained adequate surveys at regular intervals and continuous observations at magnetic observatories.

§ 3. GEOMAGNETIC SURVEYS

The accuracy of magnetic observations depends mainly on the type and character of equipment, plan and technique of operation, and skill and care of the observer. Thus measurements of a half-century ago are less accurate than those made in more recent years because of improved and superior instruments and techniques. An important illustration of this trend was the introduction of the earth-inductor for measuring magnetic dip to supplant the dip-circles of earlier generations, with substantial increase in accuracy. Loss of accuracy may result from deterioration of an instrument after years of use. The observational programme and the number of measurements required at a station vary with each data-gathering organization. Although procedures of measurements are well standardized on the whole, the results obtained by different observers show small differences when using the same instrument.

There are many sources of error: for example, magnetic impurities in the metal parts of an instrument; changes with time in adopted standards, such as change in the moment of inertia of the oscillating magnet-system in measuring

horizontal intensity; changes with time in deflection distances; wear in bearings and brushes; unspecified changes in instrumental constants not readily ascertained but detected by inter-comparisons with carefully maintained standard instruments; poor levelling and influences of temperature-changes during measurements; infrequent inter-comparisons of magnetic instruments with reliable standards; and faulty computations of results—the last often very serious. Fortunately, in most surveys, errors from these sources are of minor character and less important than the influences of extraneous fluctuations on data used in mapping the main field and its secular vaiation.

The large portion of the Earth's surface covered by the oceans makes the determination of accurate values at sea a major objective of the world-wide magnetic survey. It was not until 1905 that full realization of this objective had a definite beginning through systematic oceanic surveys then sponsored by the Carnegie Institution of Washington.

As you are well aware, the first attempt to accomplish a magnetic survey at sea was the expedition of Halley between 1698 and 1700 (Chapman and Bartels, 1940; Fleming, 1940). He made several voyages in the North and South Atlantic oceans, determining magnetic declination only. The results were embodied in his chart "Lines of equal magnetic variation" of the Atlantic for the year 1700—the first isomagnetic chart. In the past 100 years extensive magnetic surveys have been made by many countries. Important undertakings following were in the *Erebus*, the *Terror*, and the *Pagoda* between 1840 and 1845, chiefly in southern waters; on these, all elements were observed utilizing the Fox dip-circle for measuring magnetic inclination and intensity. The frigate *Novara* obtained magnetic declinations while circumnavigating the globe in 1857–60. Observations of the three magnetic elements were made over various oceans during the notable cruises of the *Challenger* in 1872–76 and of the *Gazelle* in 1874–76. More recently, observations were also made by the naval services of the various countries and by antarctic expeditions, notably the *Discovery* and the *Gauss*. The accompanying charts (figures 6*a*, 6*b*, 6*c*) show the tracks of the chief vessels on which magnetic observations were made from 1839 to 1916.

All these observations were of varying degrees of accuracy set by available instruments and by the disturbing factors originating in the magnetic character of the vessels. The distribution, both as regards position and epoch, was not such as to yield coordinated charts applying to definite epochs. Therefore, in planning in 1904 to fill in the gaps in data for the remaining regions on land and sea for the world magnetic survey, the Carnegie Institution of Washington gave careful consideration to the oceanic survey. The Institution's earliest work was with the chartered vessel *Galilee* in the Pacific Ocean during 1905 to 1908. The *Carnegie* was designed in 1908 primarily for magnetic surveys and investigations, and the vessel and equipment were completed in 1909. Before the loss of the *Carnegie* at Apia in November, 1929, the ten cruises of that vessel and the *Galilee* had traversed some 362,000 nautical miles and from tens of thousands of individual observations had obtained values of declination at over 3800 stations and of both inclination and horizontal intensity at over 2300 stations. Figure 7*a* shows in a general way the scope of these activities from 1905, whilst figure 7*b* shows a chart of horizontal intensity obtained from the existing sources of information in 1940.

On the side of practical application, the increasing use of the oceans in the commerce of nations by sea and air makes the continuation of the survey a matter of international concern and benefit. First among those investigations demanding

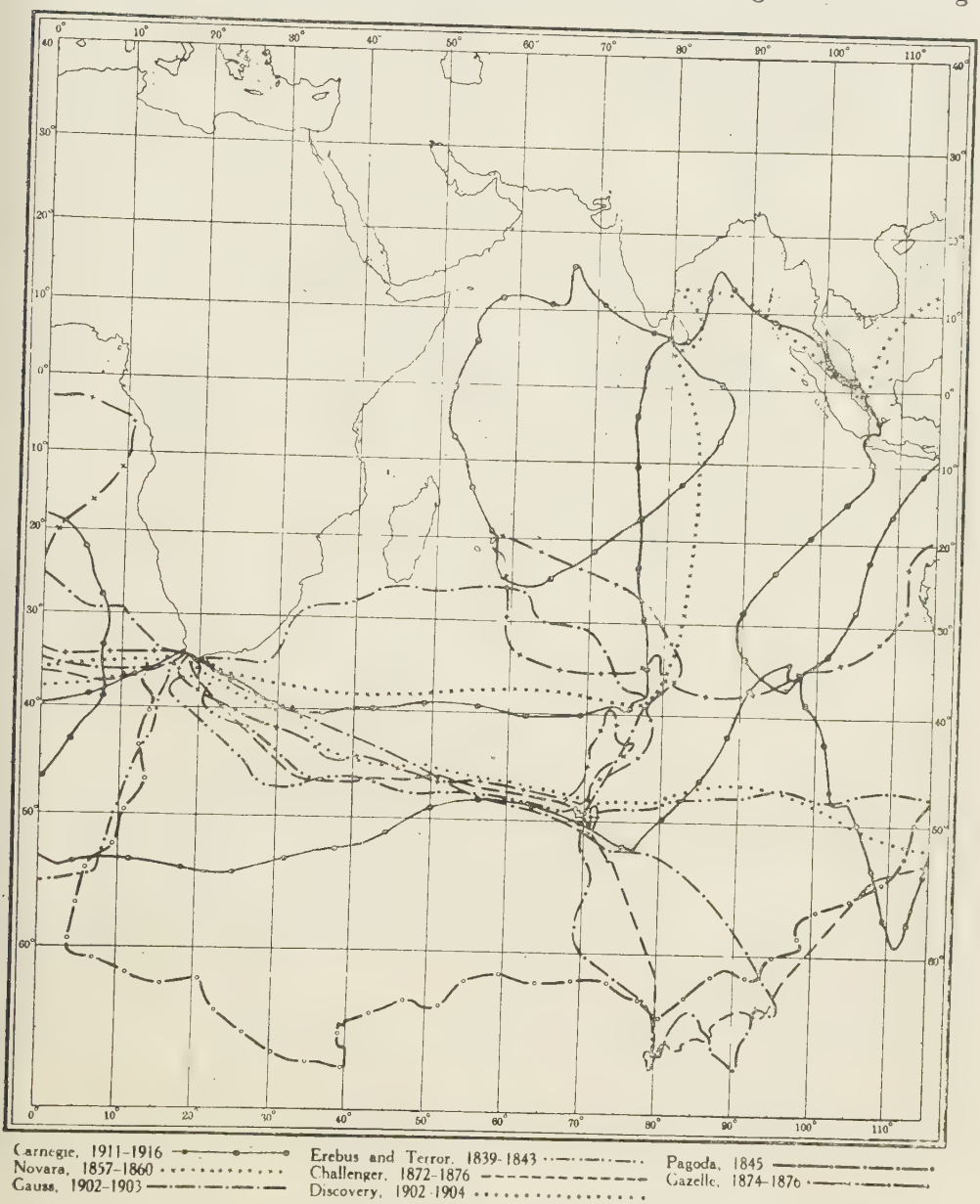

Figure 6 *a*. Tracks of chief vessels on which magnetic observations were made in the Indian Ocean, 1839–1916.

continuation of the surface magnetic survey is the determination of the acceleration of secular variations; accumulated data show that they cannot be extrapolated reliably over periods as long as five years. A definite control is necessary for a number of epochs to facilitate the investigation of causes producing and governing

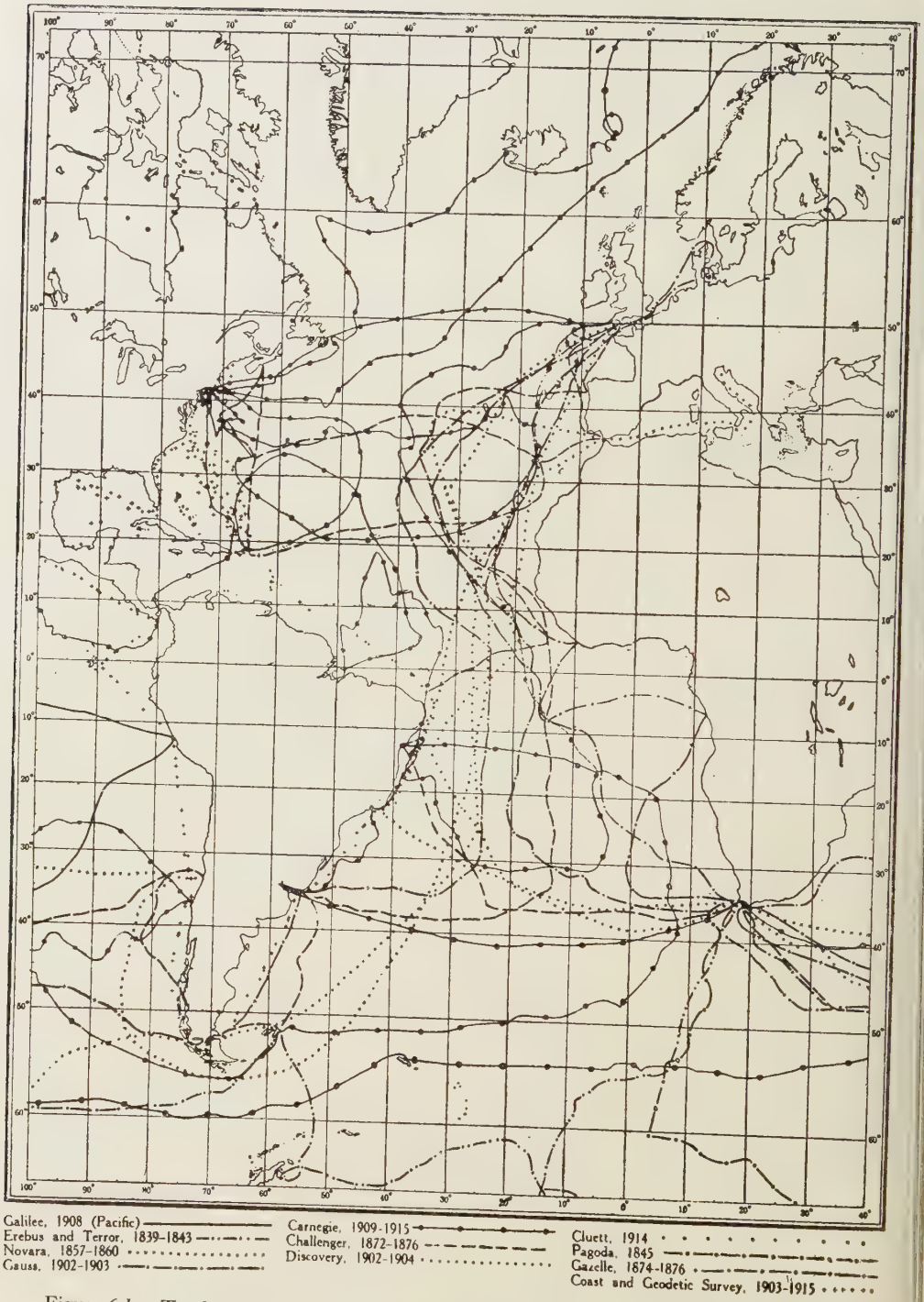

Figure 6 *b*. Tracks of chief vessels on which magnetic observations were made in the Atlantic Ocean, 1839–1916.

these progressive changes, which, it appears, would be favoured by accurate knowledge of their accelerations and distribution. The observations at repeat-stations on land have helped greatly, in conjunction with more detailed results of magnetic observatories, in delineating the complex patterns of secular change on

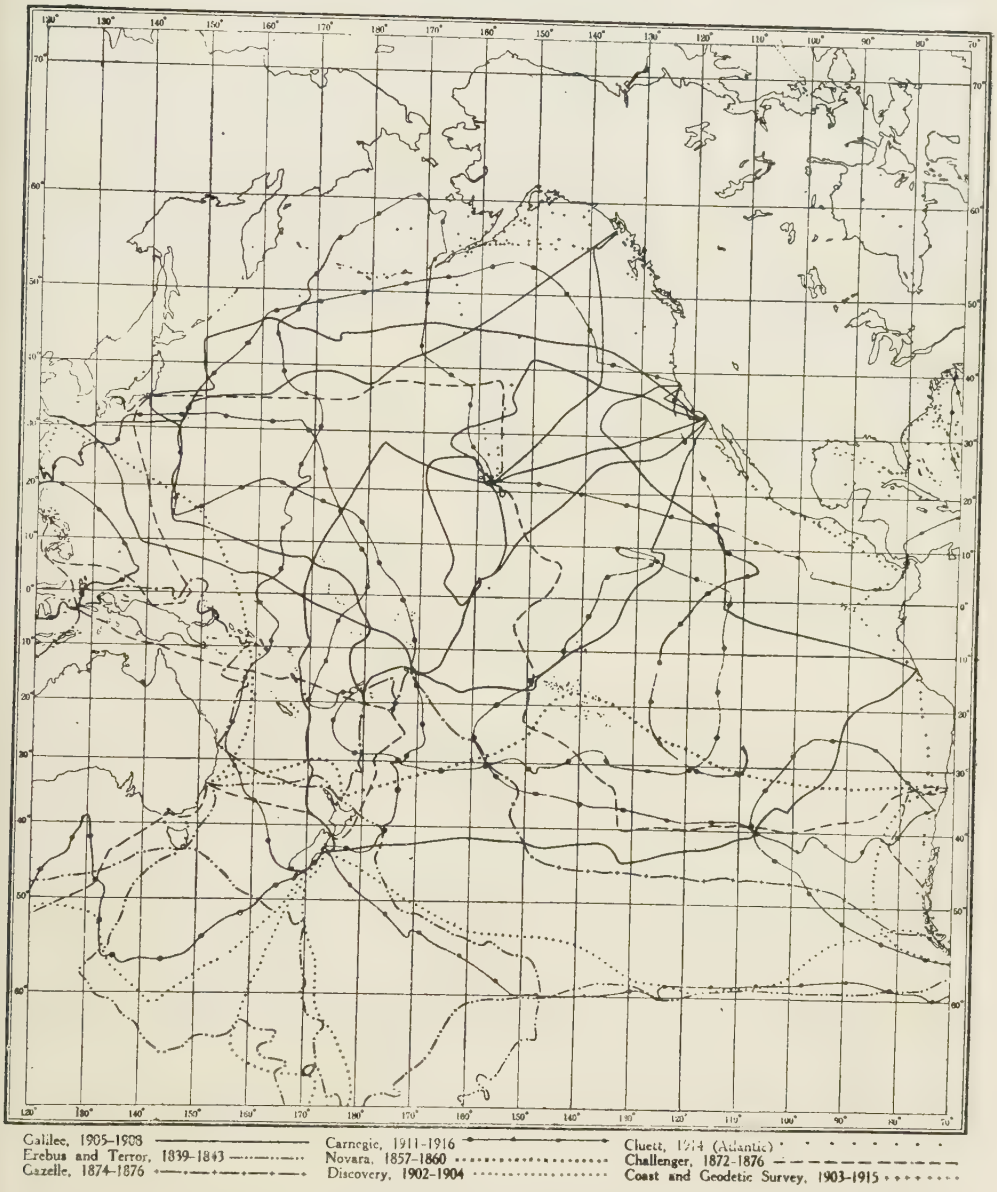

Figure 6 *c*. Tracks of chief vessels on which magnetic observations were made in the Pacific Ocean, 1839–1916.

land. By continuity these have served in some uncertain measure to bridge the gap across the oceans as well, when supported by the secular changes given by the coarser and less accurate ocean observations. It should be noted here that, though the cruises of the *Galilee* and *Carnegie* afforded usually only rough indications of

secular change in field, there were discovered great discrepancies in existing charts of the field itself. Great gaps in information remain in polar regions; for example,

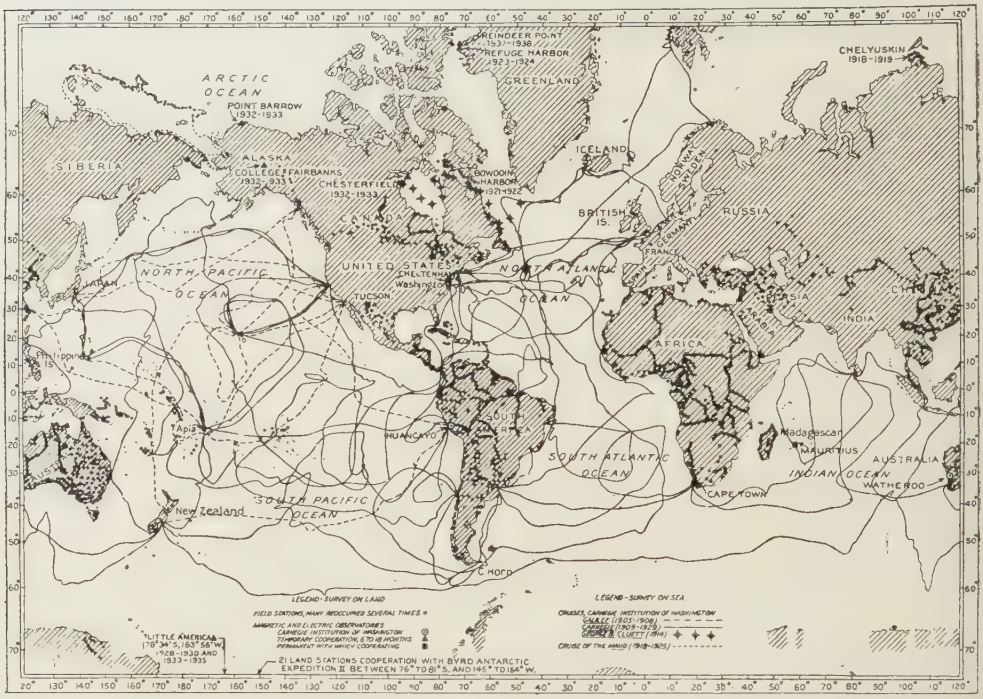

Figure 7 *a*. Magnetic survey operations of Department of Terrestrial Magnetism, Carnegie Institution of Washington, from 1905.

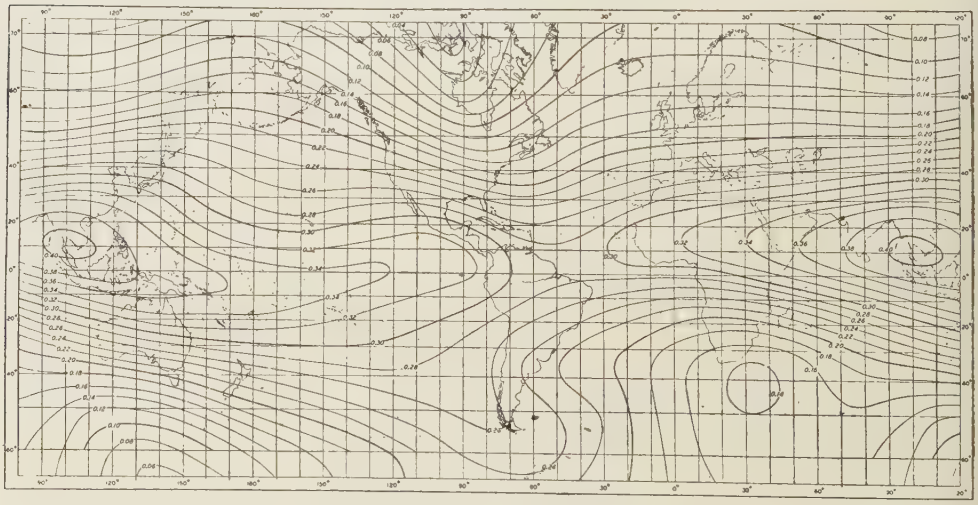

Figure 7 *b*. Isomagnetic chart of horizontal intensity of the earth's magnetic force for the year 1940 (expressed in centimetre-gram-second units).

the south polar regions have only about four badly distributed stations yielding inferior indication of secular change since 1900.

It is gratifying that the British Admiralty designed and constructed the non-

magnetic vessel *Research* to continue the surface survey at sea and to fill in those serious gaps in the present data caused by the rapid change in the secular variation in certain regions of the Earth since the earlier surveys. But the task in the geophysical survey of the oceans is so great that other hydrographic services of maritime nations should also provide suitable surface or air vessels with equipment and personnel to take appropriate and coordinated share in the world survey.

§ 4. ISOPORIC CHARTS AND GEOLOGIC FEATURES

That irregularities of the distribution of the Earth's magnetic field over continental areas may bear definite relation to geologic features of the Earth's crust has been recognized for a long time. It is only in comparatively recent years that such magnetic data have been intensively considered as an aid in the determination of the character of geologic substructure. From accumulated observations it is possible to account for a considerable part of the Earth's main field on the basis of a uniformly magnetized sphere. The difference between such a uniform or normal magnetization and that actually existing serves to bring out the general irregularities as a residual field. The Earth's outer crust or shell, perhaps 25 or more miles thick, is not homogeneous and, therefore, is not uniform in its magnetic behaviour. Thus there are many regions of local magnetic disturbance, such as are caused by magnetic ore-deposits, some so great as to give to local poles and other irregularities of intensity twofold or even threefold the normal value in such regions. Anomalies in Russia, in Germany, and in New South Wales are a few striking examples of such magnetic irregularities. The great anomaly in South Africa has been apparently retained for over 100 million years by the huge Pilansberg system of Palæozoic volcanic dykes (figure 8); fourteen vertical dykes are found to be strongly magnetized in a direction opposite to the present-day magnetic field of the Earth (Piggot, 1938). In this general region of South Africa, the magnetic horizontal component of the Earth's field has decreased 16% of its present value in a 30-year period!

The anomalous magnetic features thus observed over continental areas must have their counterparts in the great oceanic basins (Fleming, 1938 b). We may therefore look confidently to such features as an aid in the study of oceanic structure. Indeed, observations made on isolated islands and island groups almost invariably indicate abnormal magnetic conditions, and the magnetic survey of the oceans has revealed such conditions at sea.

However, the application of magnetic-survey results to the investigation of crustal features of the ocean bottom depends upon the existence of a difference in substructural physical properties as well as upon the magnitude of this difference. Here enters the magnetic susceptibility of rocks, some of which are strongly magnetic, as iron-ores and igneous and metamorphic rocks, and some weakly magnetic, as most sedimentary rocks. Here, then, one must think of the observed magnetic anomalies as a kind of general reconnaissance method to indicate areas for other more intensive geophysical studies. However, the magnetic method of approach lacks depth-control, and, therefore, the size of geologic features must increase in proportion to depth if we are to detect such features. Thus, unless the magnetic anomaly observed at sea is over shallow water or, if over deep water, is of enormous proportions, there may not be great hope of immediately deriving exact

information regarding particular areas of bottom substructure from distribution data as shown only by isomagnetic maps.

The secular-variation change of the Earth's magnetic field is a phenomenon which has large potential value in the study of the Earth's crust. Herein is the great need of continuance of the accurate world surface magnetic survey, that we may determine the character of such changes with epoch and their accelerations at various places.

Ever since the discovery of the change in the Earth's magnetic field through the centuries, investigators have noted the apparent symmetry and regularity of secular variation when attention is confined to a single station or to a small part of

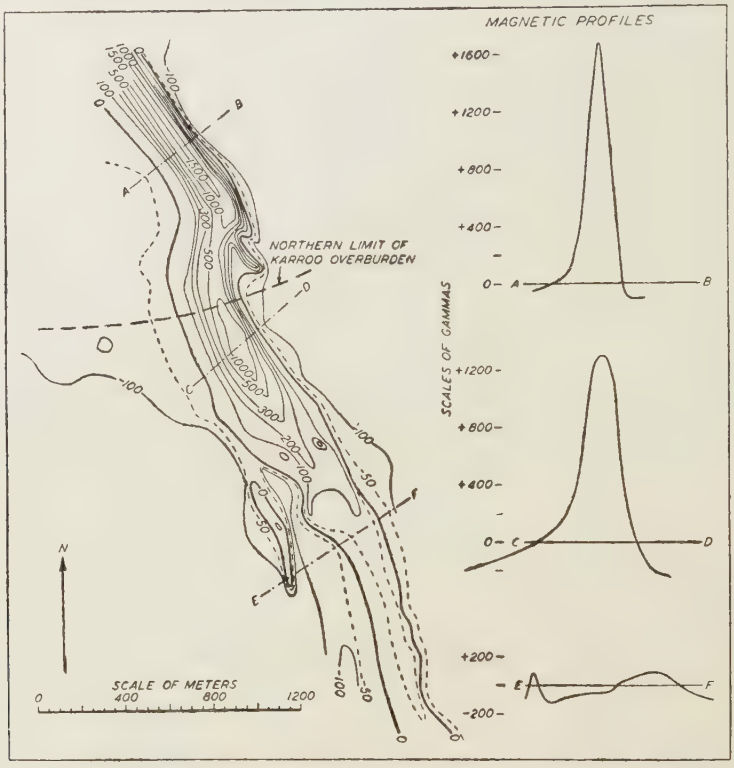

Figure 8. Magnetic anomaly of Pilansberg Dyke System, southern Transvaal, showing anomaly of vertical intensity expressed in gammas (after Gelletich).

the Earth's surface such as western Europe—only one-hundredth of the Earth's surface. Considering, however, the actual observational data when the whole Earth is taken into account, secular variation is not so simple a phenomenon that it may be explained by periodic movements of the Earth's magnetic poles and the like, but is a regional phenomenon connected with a large geological structure—oceans and continents—of the deeper layers of the Earth's crust.

The foci of rapid isoporic changes raise questions of broad geophysical significance. A large part of these changes may have origin in influences impressed by forces ceaselessly at work within the Earth. The rise and fall of the rate of secular change and the slow expansion and then the gradual retraction of the areas within which there have been excessive alterations in the magnetic elements are

significant of such changes. The distribution of isoporic foci (figures 9 *a*, 9 *b*) is practically all in the hemisphere containing the great land-masses with the intervening Atlantic Ocean. Such foci as are found in the Pacific Ocean are of moderate intensity and not well defined. These relations to the surface structure of the Earth can be scarcely accidental; it is natural to suspect that there is a causal relation between crustal or subcrustal movements and conditions and these magnetic manifestations.

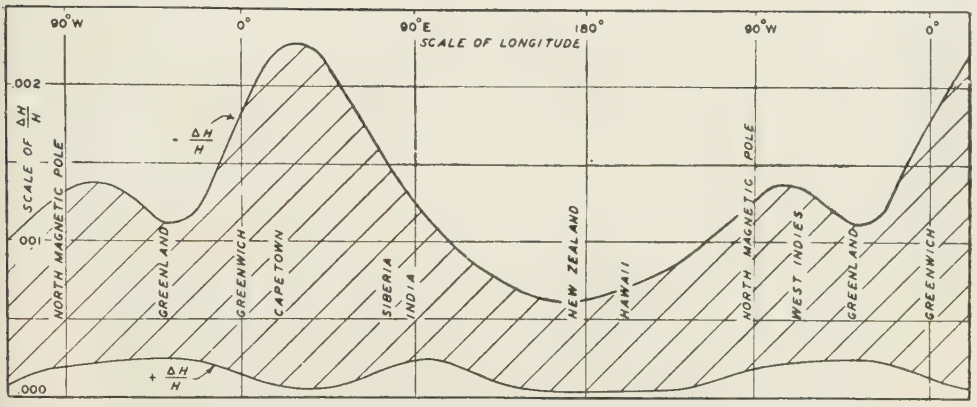

Figure 9 *a*. Longitudinal distribution of proportion of annual change ($\Delta H/H$) of horizontal intensity (after Fisk). (Lower curve represents average positive values of $\Delta H/H$ in each lune between meridians, while upper curve represents numerical magnitude of average negative values ; thus shaded areas between curves are measures of excess of negative over positive annual change.)

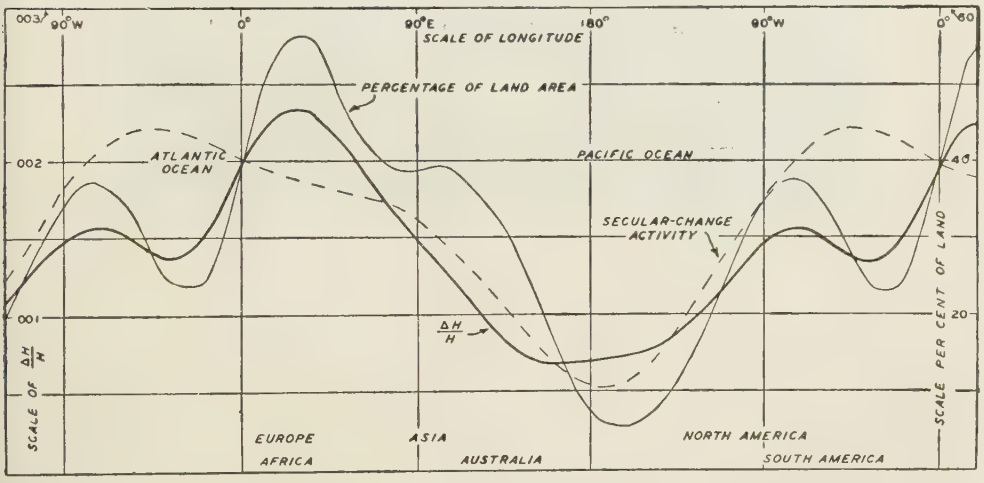

Figure 9 *b*. Variation with longitude of $\Delta H/H$ (annual change averaged without regard to sign), of distribution of proportion of land and water areas, and of secular-variation activity approximately determined by density of distribution of isoporic lines (after Fisk).

As an example, attention may be called to the apparent diminution of the intensity of the Earth's magnetic field which is marked over oceanic areas, especially in the Southern Hemisphere (Fisk, 1932). The interpretation of such data must be important in geophysical and geological research to advance understanding of Earth phenomena.

Thus the observed longitudinal distribution of magnetic secular variation agrees with that of land areas— as witness the moderate rates of annual change over the Pacific as compared with those over the Atlantic and adjoining continental areas. A definite control is necessary for a number of epochs to facilitate investigation of causes producing and governing these progressive changes.

Further data bearing on correlation between the surface-distribution of secular-change activity promise conclusions concerning isoporic processes localized in the crustal layer. Thus continued secular-variation surveys at sea should bring together seismic and magnetic methods of approach to crustal adjustments and possibly gravimetric work.

Secular-variation information may be extended to remote times for which there are no observations. When lava cools and solidifies, following a volcanic outburst, it takes up a permanent magnetization dependent upon the orientation of the Earth's magnetic field at the time. This, because of small capacity for magnetization in the Earth's magnetic field after solidifying, may remain practically

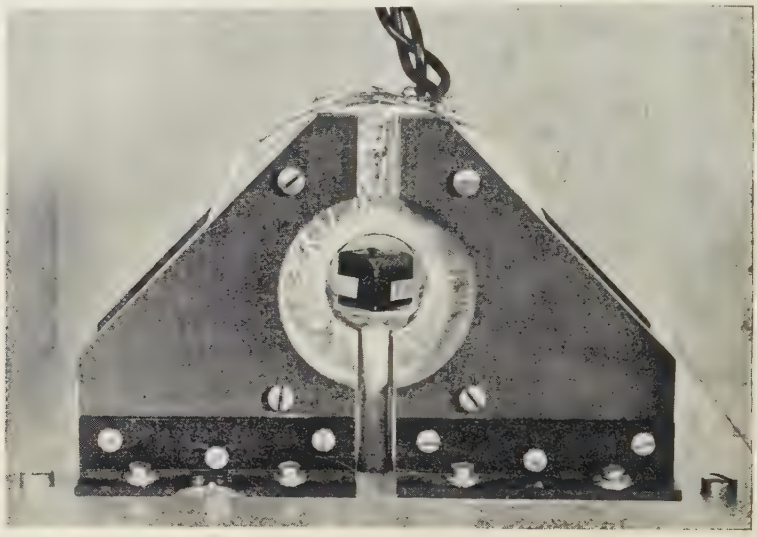

Figure 10. Apparatus for measuring magnetic moments and orientations of samples of varves and of ocean-bottom cores.

constant. Thus the direction of the originally acquired permanent magnetization can be determined by laboratory tests provided every detail of the orientation of the mass tested is carefully noted and marked when it is removed.

Just as lavas cooling through the Curie point in the presence of a magnetic field become magnetized in the direction of the field, so sediments containing microscopic magnetic particles also assume the direction of the prevailing magnetic field. This is truly "fossil" magnetization, for the magnetic field of the epoch of deposition is embedded in the sediment as surely as the organic remains of that epoch. Whether this fossil magnetism remains as a connate property of the rock or whether it is destroyed by subsequent events, even as organic fossils are sometimes destroyed, depends upon the post-natal experience of the sediment. In so far as it remains, it may supply an important record of the Earth's magnetism and an important clue to the history of the rock.

The measurement of the Earth's magnetism at great depths in the sea may now be consummated by magnetic examination of the core-samples taken with Dr. Piggot's apparatus described in 1938. In order that the fossil magnetization of the specimens may not be disturbed, a method of measurement has been developed for tests of high sensitivity and accuracy without submitting the specimens to

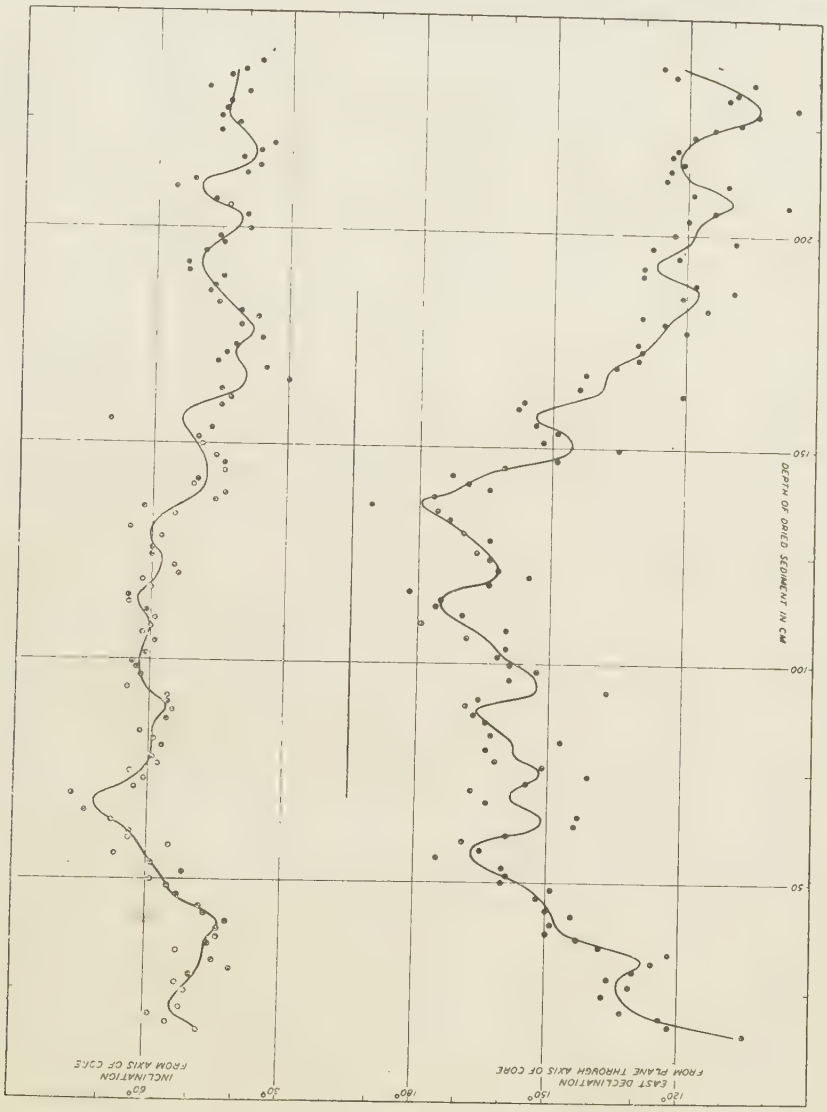

Figure 11. Direction of magnetization of sediments in core No. 3 (curve smoothed by formula $(a+2b+3c+2d+e)/9$).

artificial magnetic fields. This equipment (figure 10) permits detection of a magnetic moment as minute as that of a good steel magnet one millimetre long and one-hundredth of a millimetre in diameter. Further refinements will increase many-fold the sensitivity of this method.

Tests on one core show distinct differences in the direction of magnetization at various depths (figure 11). If it is found that these differences are the result of

differences in the Earth's field at the time the sediments were deposited, we have in the core-samples a means of extending the past history of geomagnetism and its secular changes. Interrelating individual cores by similarities in their magnetism would determine more accurately than surface observations the extent and character of the regional magnetic anomalies of the ocean bottom.

With improved knowledge of magnetic anomalies, the question of interrelations with gravimetric anomalies and deep-focus earthquakes may be answered. It is frequently noted that gravimetric and magnetic anomalies occur in the same regions. Apparently areas of deep-focus earthquakes are confined to the great regions of positive anomalies of magnetic vertical intensity around the Pacific Ocean. Vening Meinesz has found an excess of gravity in the oceans and a sudden decrease at the continental borders, and has demonstrated that these deviations of isostasy cannot be situated in the Earth's thin crust but must extend far into the plastic layers below. There must result mass flow in the Earth's interior, downward below the oceans and upward below the continents. The charts of isoporic foci and their motions with time certainly lend strength to the idea, indicating the interior of the Earth to be more mobile than the external layers, not only as a whole but regionally. May not, then, the anomalies of gravity and of geomagnetism and the deep earthquakes have a common origin in mass flows in the inner Earth? Thus the facts of the geophysical observer of geomagnetism in the field and in the laboratory may contribute much to a complete picture of the Earth's crust and particularly of that great portion which is the ocean bottom.

§ 5. RECENT DERIVATIONS OF SECULAR CHANGE

The Department of Terrestrial Magnetism has compiled isoporic charts of the geomagnetic field for the four epochs of 1912·5, 1922·5, 1932·5, and 1942·5 for declination, inclination, and the northward, eastward and total components of intensity. These were compiled preparatory to the construction of new world isomagnetic charts of these elements in seventeen sections for the epoch 1945·0.

All available repeat-station observations have been used in compiling secular change for each of the seven elements and components. All have been corrected for the influence of extraneous geomagnetic variations, including those related to the sunspot cycle (not properly considered as secular change), post-perturbation, annual variation, and the quiet-day solar daily variation. Graphs of the observed values so reduced to "normal" were made for each station. Specimen graphs for the station Suez and the Alibag Observatory are reproduced in figures 12 (*a*, *b*, *c*) and 13 (*a* and *b*). The values as actually observed are plotted in order to show the order of correction required to obtain reduced "normal" values. The corrections at Suez are largest for the horizontal and northward components, due to the considerable latitude-effect of the geomagnetic variation with sunspot-cycle and of post-perturbation. The corrections are of greatest percentage benefit when the secular changes are small. The same considerations apply also in the case of observatories such as Alibag; here the corrections (figures 14 *a* and 14 *b*), as for many series of data, have been of material assistence in improving estimates of secular change in intensity near 1910.

Graphs of the foregoing kind were scaled at epochs 1912·5, 1922·5, 1932·5, and 1942·5, and plotted on world maps. A sample of these is given in figures 15 (*a*, *b*, *c*) and 16 (*a*, *b*, *c*), showing the isopors in declination and horizontal

[*Continued on p.* 240.

ESTIMATED GEOMAGNETIC DEPARTURES, INCLUDING YEARLY MEAN OF HOUR (MH) FROM MEAN OF YEAR (MY_0) DEPARTURE OWING TO VARIATION WITH SUNSPOT-CYCLE (RV) CENTERED ABOUT MEAN OF MONTH (MM_0)

STATION: Suez, C COUNTRY: Egypt, Africa

FORMULAS: $\Delta D = A_D \Delta X' + B_D \Delta Y'$; $\Delta H = A_H \Delta X' + B_H \Delta Y'$; $\Delta I = A_I \Delta H_I + B_I \Delta Z_I$;

$A_D = -.026$ (25), $B_D = .111$ (26), $A_H = .974$ (27), $B_H = .227$ (28), $A_I = -.057$ (29), $B_I = .063$ (30)

Element (1)	Date and local mean time (2)	C (3)	Geomagnetic north component, $\Delta X'$: For S_q (4)=(45)	For S_D (5)	For D_{st} (6)	For D_i (7)	(8)	For MM_0 to MY_0 (9)=(54)	For MD to MM_0 (10)=(57)	For RV (11)	Total departure on $MY_0 = \Sigma$ (12)	Geomagnetic east component, $\Delta Y'$: MH to MD: For S_q (13)=(69)	For S_D (14)	For D_{st} (15)	For D_i (16)	(17)	For MH to $MY_0 = \Sigma$ (18)	Product (19)	Product (20)	$\Delta H_I = \Sigma$ (21)	For MH to $MY_0 = \Sigma$ (22)	Values of elements for epoch Y_0 (1935.367) = [(Observed IMS values at station) − (22)] (72)
D	1935 h		γ	γ	γ	γ	γ	′	γ	γ	(4) to (11) γ	γ	γ	γ	γ	γ	(13) to (16) γ	(25) x (12) ′	(26) x (18) ′		(19) + (20) ′	
	May 23 14.4	0.1	+24	0	0	0	...	+8	−6	+7	+33	−21	0	0	0	...	−21	−.9	−2.3		−3.2	D = +0 13.0
	16.2	0.1	+11	0	0	0	...	+8	−6	+7	+20	−8	0	0	0	...	−8	−.5	−0.9		−1.4	cos D = +.99999
	May 24 9.8	0.0	−9	0	0	0	...	+8	0	+7	+6	+10	0	0	0	...	+10	−.2	+1.1		+0.9	sin D = +.00378
	11.4	0.0	+4	0	0	0	...	+8	0	+7	+19	−14	0	0	0	...	−14	−.5	−1.6		−2.1	
																					−1.4	
I			γ	γ	γ	γ	γ	γ	γ	γ	(4) to (11) γ	γ	γ	γ	γ	γ	(13) to (16) γ	(27) x (12) γ	(28) x (18) γ	(19) + (20) γ	(23) + (24) ′	° ′
	May 23 13.8	0.1	+25	0	0	0	...	+8	−6	+7	+34	−24	0	0	0	...	−24	+33.1	−5.4	+27.7	−2.0	I = +42 01.5
	May 24 9.0	0.0	−14	0	0	0	...	+8	−1	+7	0	+20	0	0	0	...	+20	0	+4.5	+4.5	−0.4	sec I = +1.34616
	12.2	0.0	+14	0	0	0	...	+8	0	+7	+29	−19	0	0	0	...	−19	+28.2	−4.3	+23.9	−2.0	tan I = +.90120
																					−1.5	
H			γ	γ	γ	γ	γ	γ	γ	γ	(4) to (11) γ	γ	γ	γ	γ	γ	(13) to (16) γ	(27) x (12) γ	(28) x (18) γ	(19) + (20) γ		
	May 23 14.9	0.1	+22	0	0	0	...	+8	−6	+7	+31	−19	0	0	0	...	−19	+30	−4	+26	H = 30360 γ	
	15.7	0.1	+16	0	0	0	...	+8	−6	+7	+25	−12	0	0	0	...	−12	+24	−3	+21	F = H sec I = 40869 γ	
	May 24 10.2	0.0	−6	0	0	0	...	+8	0	+7	+9	+4	0	0	0	...	+4	+9	+1	+10	Z = H tan I = 27360 γ	
	11.0	0.0	0	0	0	0	...	+8	0	+7	+15	−10	0	0	0	...	−10	+15	−2	+13	X = H cos D = 30360 γ	
																				+18	Y = H sin D = +115 γ	
																					X^2 = 921729600	
																					Y^2 = 13225	
																					Z^2 = 748569600	
																					F^2 = 1670275161	

Magnetic vertical component, ΔZ											Component ΔI: $A_I \Delta H_I$ (23)	$B_I \Delta Z_I$ (24)
Z			γ	γ	γ	γ	γ	γ	γ	γ	(4) to (11) γ	(29) x (21) ′	(30) x (12) ′
	May 23 13.8	0.1	−6	0	0	0	...	0	+1	−1	−6	−1.6	−.4
	May 24 9.0	0.0	0	0	0	0	...	0	0	−1	−1	−0.3	−.1
	12.2	0.0	−9	0	0	0	...	0	0	−1	−10	−1.4	−.6

LEGEND

C = International magnetic character-figure

S_q = Solar daily variation

S_D = Disturbance daily variation

D_{st} = Storm-time variation

D_i = Irregular disturbance

RV = Geomagnetic variation yearly means with sunspot-cycle

Enter components magnetic intensity to nearest γ, D and I to nearest 0.1′

Entries for $\Delta X'$, $\Delta Y'$, ΔH_I, and ΔZ_I against elements I and Z when Z is not observed are for times of observation of I

Figure 12 *a*. Specimen of estimated geomagnetic departures, mean of hour (MH) from mean of year (MY_0) centred about mean of month (MM_0), including yearly departure owing to variation with sunspot-cycle (RV) for Suez, station C of 1935.

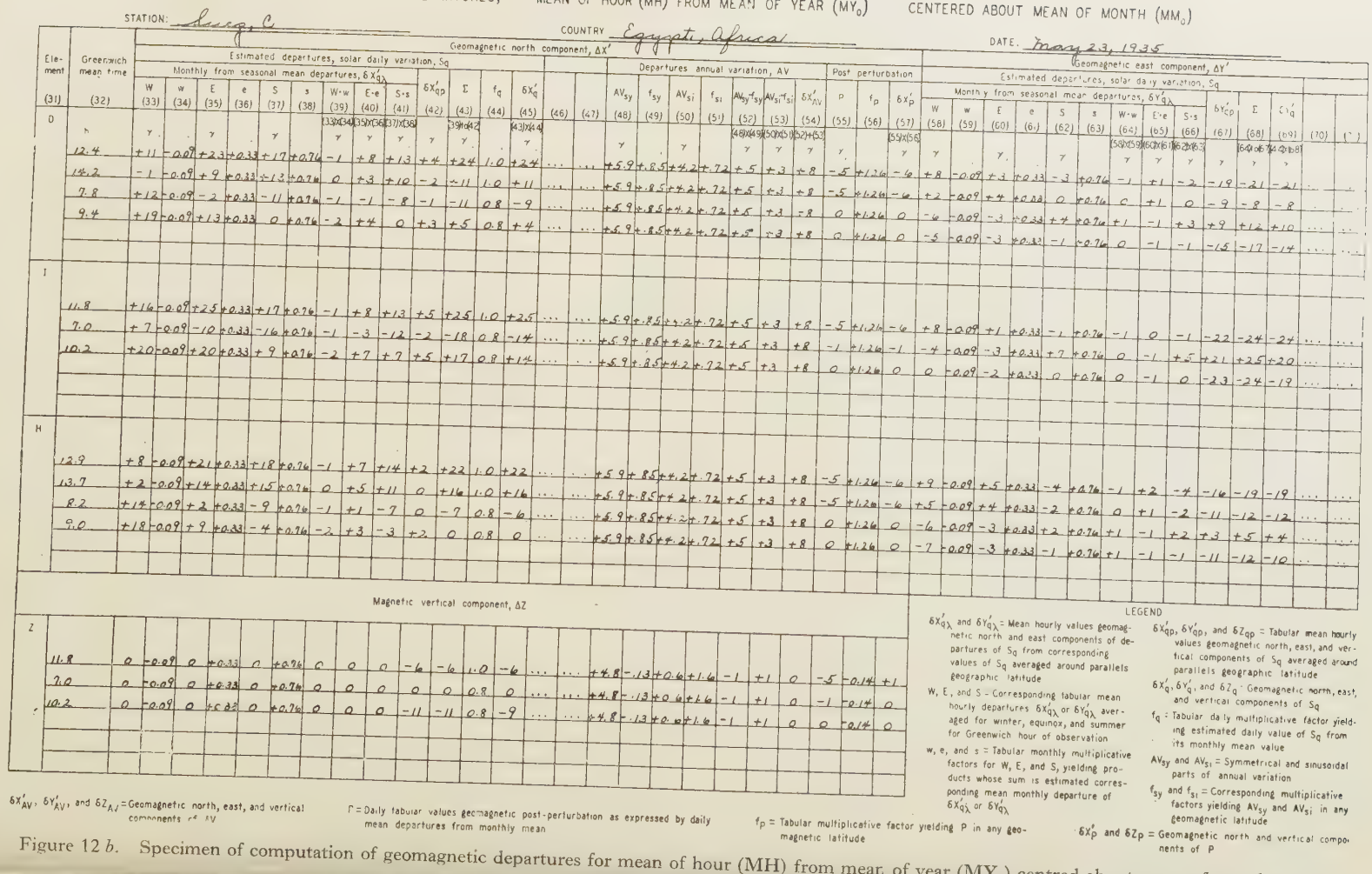
COMPUTATION OF GEOMAGNETIC DEPARTURES, MEAN OF HOUR (MH) FROM MEAN OF YEAR (MY$_0$) CENTERED ABOUT MEAN OF MONTH (MM$_0$)
STATION: Sitka, C.
COUNTRY Egypt, Africa
DATE. May 23, 1935
Element (31)
Greenwich mean time (32)
Estimated departures, solar daily variation, Sq
Geomagnetic north component, ΔX′
Monthly from seasonal mean departures, δX′$_{Sq}$
W (33) w (34) E (35) e (36) S (37) s (38)
W-w (39) E-e (40) S-s (41)
δX′$_{qp}$ (42) Σ (43) f$_q$ (44) δX′$_q$ (45)
Departures annual variation, ΔV
AV$_{sy}$ (48) f$_{sy}$ (49) AV$_{si}$ (50) f$_{si}$ (51) AV$_{sy}$f$_{sy}$ (52) AV$_{si}$f$_{si}$ (53) δX′$_{ΔV}$ (54)
Post perturbation
P (55) f$_p$ (56) δX′$_p$ (57)
Geomagnetic east component, ΔY′
Estimated departures, solar daily variation, Sq
Monthly from seasonal mean departures, δY′$_{Sq}$
W (58) w (59) E (60) e (61) S (62) s (63)
W-w (64) E-e (65) S-s (66)
δY′$_{qp}$ (67) Σ (68) f′$_q$ (69)
Magnetic vertical component, ΔZ
Z
δX′$_{AV}$, δY′$_{AV}$, and δZ$_{AV}$ = Geomagnetic north, east, and vertical components of ΔV
P = Daily tabular values geomagnetic post-perturbation as expressed by daily mean departures from monthly mean
f$_p$ = Tabular multiplicative factor yielding P in any geomagnetic latitude
LEGEND
δX′$_{qλ}$ and δY′$_{qλ}$ = Mean hourly values geomagnetic north and east components of departures of Sq from corresponding values of Sq averaged around parallels geographic latitude
W, E, and S = Corresponding tabular mean hourly departures δX′$_{qλ}$ or δY′$_{qλ}$ averaged for winter, equinox, and summer for Greenwich hour of observation
w, e, and s = Tabular monthly multiplicative factors for W, E, and S, yielding products whose sum is estimated corresponding mean monthly departure of δX′$_{qλ}$ or δY′$_{qλ}$
δX′$_{qp}$, δY′$_{qp}$, and δZ$_{qp}$ = Tabular mean hourly values geomagnetic north, east, and vertical components of Sq averaged around parallels geographic latitude
δX′$_q$, δY′$_q$, and δZ$_q$ = Geomagnetic north, east, and vertical components of Sq
f$_q$ = Tabular daily multiplicative factor yielding estimated daily value of Sq from its monthly mean value
AV$_{sy}$ and AV$_{si}$ = Symmetrical and sinusoidal parts of annual variation
f$_{sy}$ and f$_{si}$ = Corresponding multiplicative factors yielding AV$_{sy}$ and AV$_{si}$ in any geomagnetic latitude
δX′$_p$ and δZ$_p$ = Geomagnetic north and vertical components of P

Figure 12 *b*. Specimen of computation of geomagnetic departures for mean of hour (MH) from mean of year (MY_0) centred about mean of month (MM_0).

MAGNETIC ELEMENTS AND THEIR SECULAR VARIATIONS BY EPOCHS

Station Suez C (new station 1935) Latitude +29° 56.4′ Computed by J.M. Date Jan. 30, 1945

Country Egypt, Africa Longitude 32° 29.0′ E Checked by R.M.B. Date Jan. 31, 1945

Element	1910		1912.5		1920		1922.5		1930		1932.5		1940		1942.5		1950		1952.5		1960		1962.5	
	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV
D	-2° 31.5′	+6.6	-2° 14.1′	+7.2	-1° 16.9′	+8.0	-0° 55.9′	+7.8	-0° 06.9′	+4.4	+0° 03.3′	+3.7	+0° 27.2′	+3.0	+0° 37.7′	+3.0								
I	+40° 39.1′	+2.9	+40° 46.5′	+3.0	+41° 12.4′	+3.7	+41° 21.8′	+3.6	+41° 46.1′	+3.1	+41° 53.7′	+3.0	+42° 14.3′	+2.7	+42° 21.1′	+2.7								
H	30202	-16	30167	-12	30121	+3	30137	+9	30243	+20	30298	+22	30471	+24	30530	+24								
X	30179	-15	30147	-11	30118	+3	30133	+9	30243	+19	30298	+22	30470	+23	30528	+23								
Y	-1319	+62	-1165	+63	-665	+70	-488	+70	-65	+41	+29	+35	+255	+26	+318	+24								
Z	25937	+34	26028	+38	26379	+56	26523	+62	27015	+67	27182	+67	27672	+63	27830	+63								
F	39804	+14	39840	+16	40032	+37	40139	+48	40559	+57	40708	+60	41160	+60	41311	+60								

Station Helwan Latitude +29° 51.6′ Computed by C.U.C. R.M.M. Date May 29, 1945

Country Egypt, Africa Longitude 31° 20.5′ E Checked by J.M. Date May 29, 1945

Element	1910		1912.5		1920		1922.5		1930		1932.5		1940		1942.5		1950		1952.5		1960		1962.5	
	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV	Value	SV
D	-2° 44.0′	+7.4	-2° 24.8′	+7.7	-1° 27.2′	+7.7	-1° 08.0′	+7.8	-0° 17.0′	+5.4	-0° 04.2′	+4.8	+0° 27.8′	+4.1	+0° 38.2′	+4.1								
I	+40° 41.2′	+1.8	+40° 46.5′	+2.5	+41° 10.2′	+3.2	+41° 18.2′	+3.2	+41° 41.0′	+3.0	+41° 47.3′	+2.6	+42° 13.7′	+4.2	+42° 24.0′	+4.2								
H	30023	-9	30006	-4	29958	-4	29956	+1	30080	+28	30150	+29	30406	+35	30490	+35								
X	29986	-4	29980	0	29948	-3	29946	+3	30077	+29	30150	+31	30403	+34	30490	+34								
Y	-1438	+64	-1261	+71	-760	+68	-592	+68	-148	+46	-041	+40	+246	+37	+340	+37								
Z	25812	+16	25880	+33	26190	+49	26319	+54	26777	+66	26942	+62	27595	+98	27840	+99								
F	39595	+3	39621	+18	39803	+27	39877	+31	40277	+45	40435	+45	41061	+93	41292	+93								

Figure 12 *c*. Summary magnetic elements and their secular variations by epochs from 1910, Suez and Helwan.

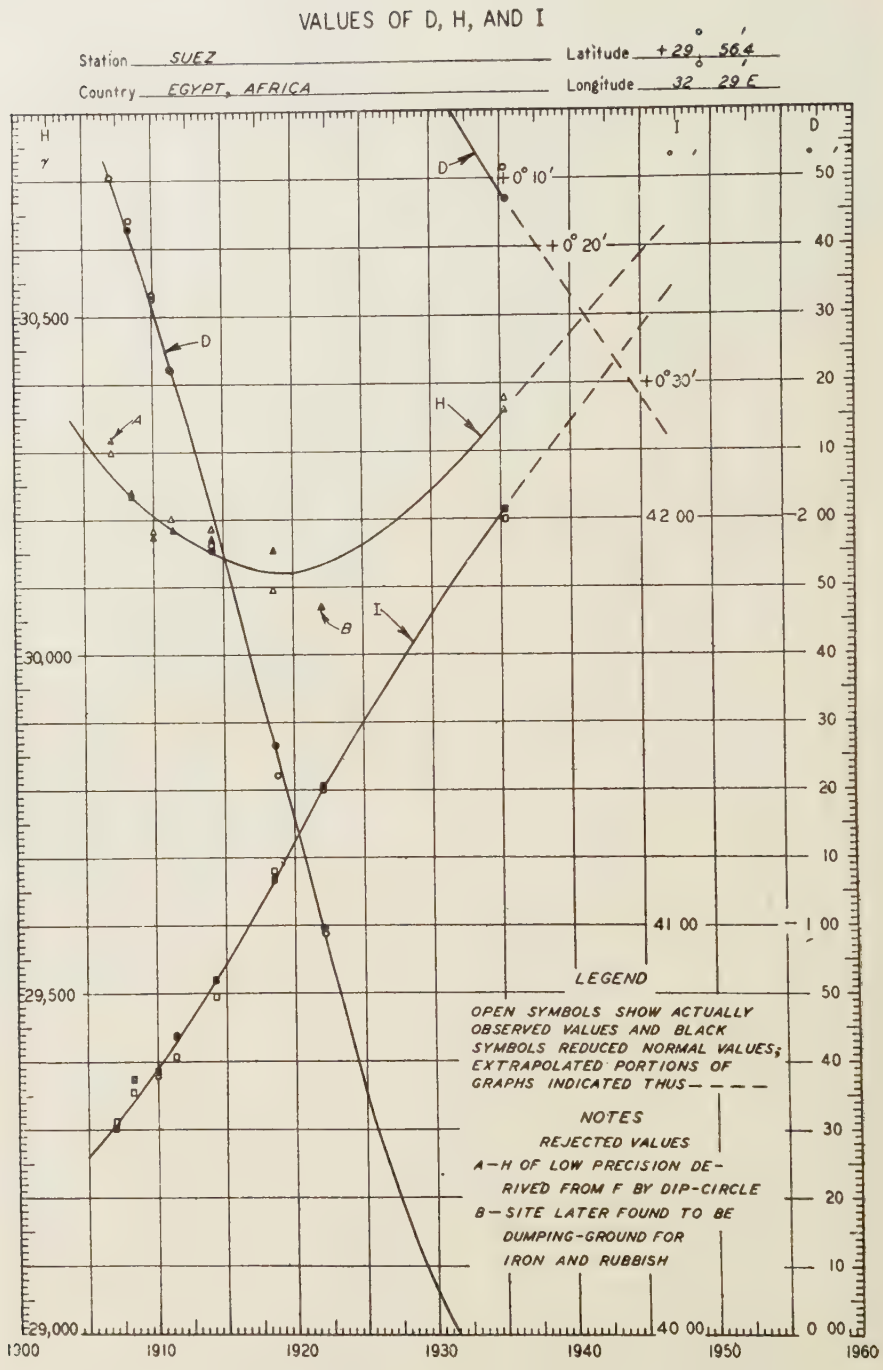

Figure 13 *a*. Specimen graphs for values of *D*, *H* and *I* from 1905, Suez.

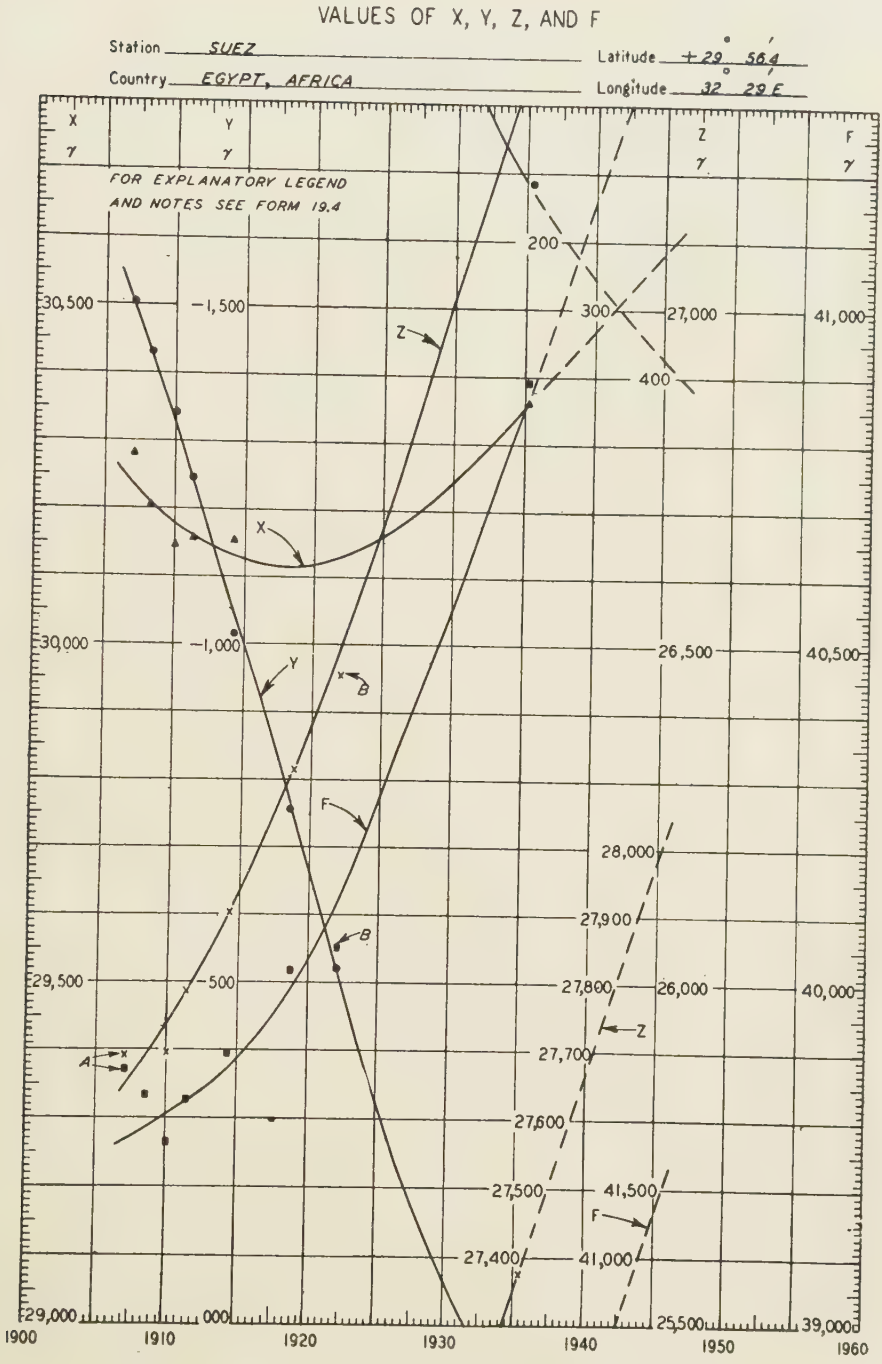

Figure 13 *b*. Specimen graphs for values of *X*, *Y*, *Z* and *F* from 1905, Suez.

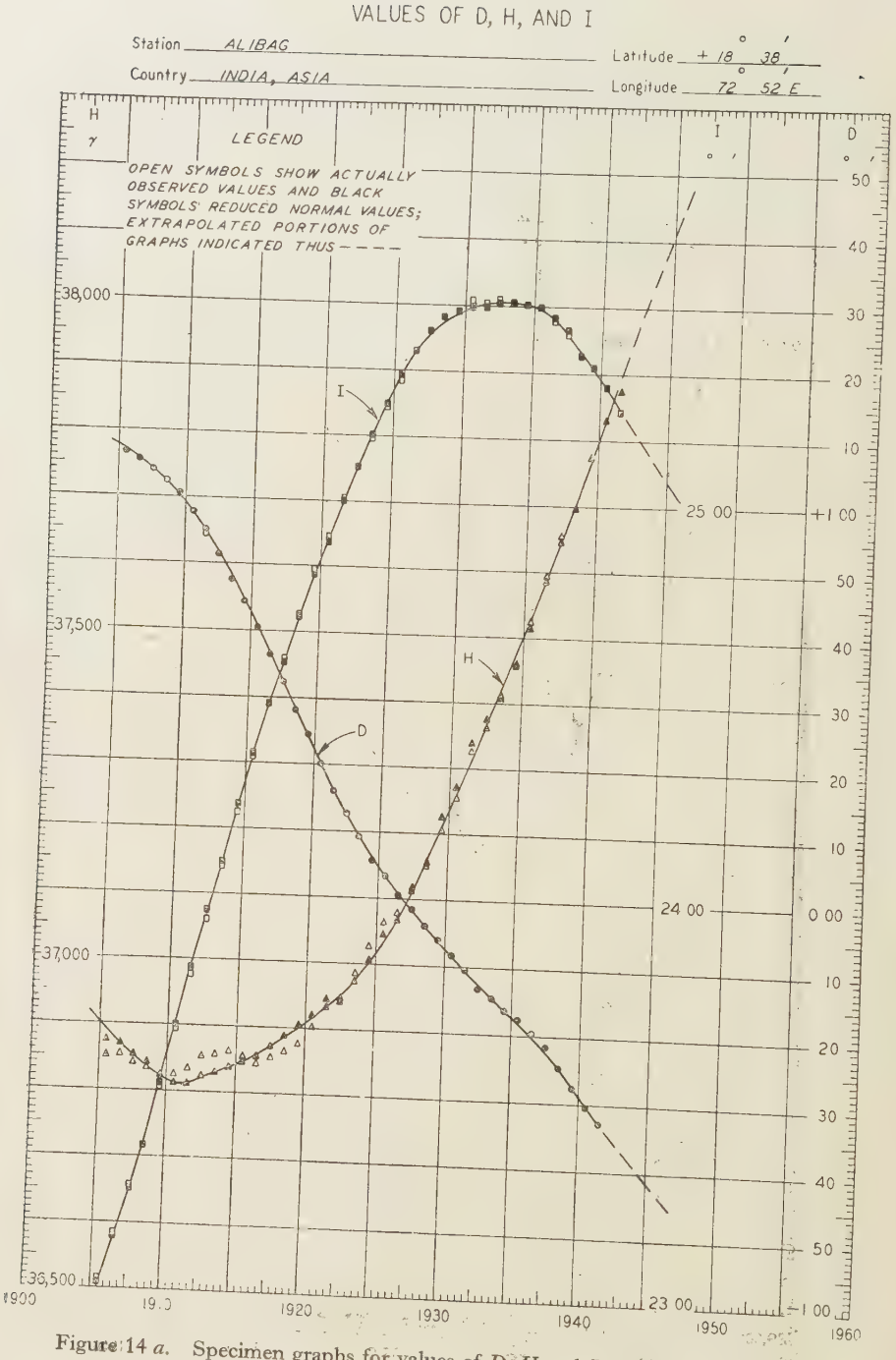

Figure 14 *a*. Specimen graphs for values of *D*, *H* and *I* from 1905 for Alibag.

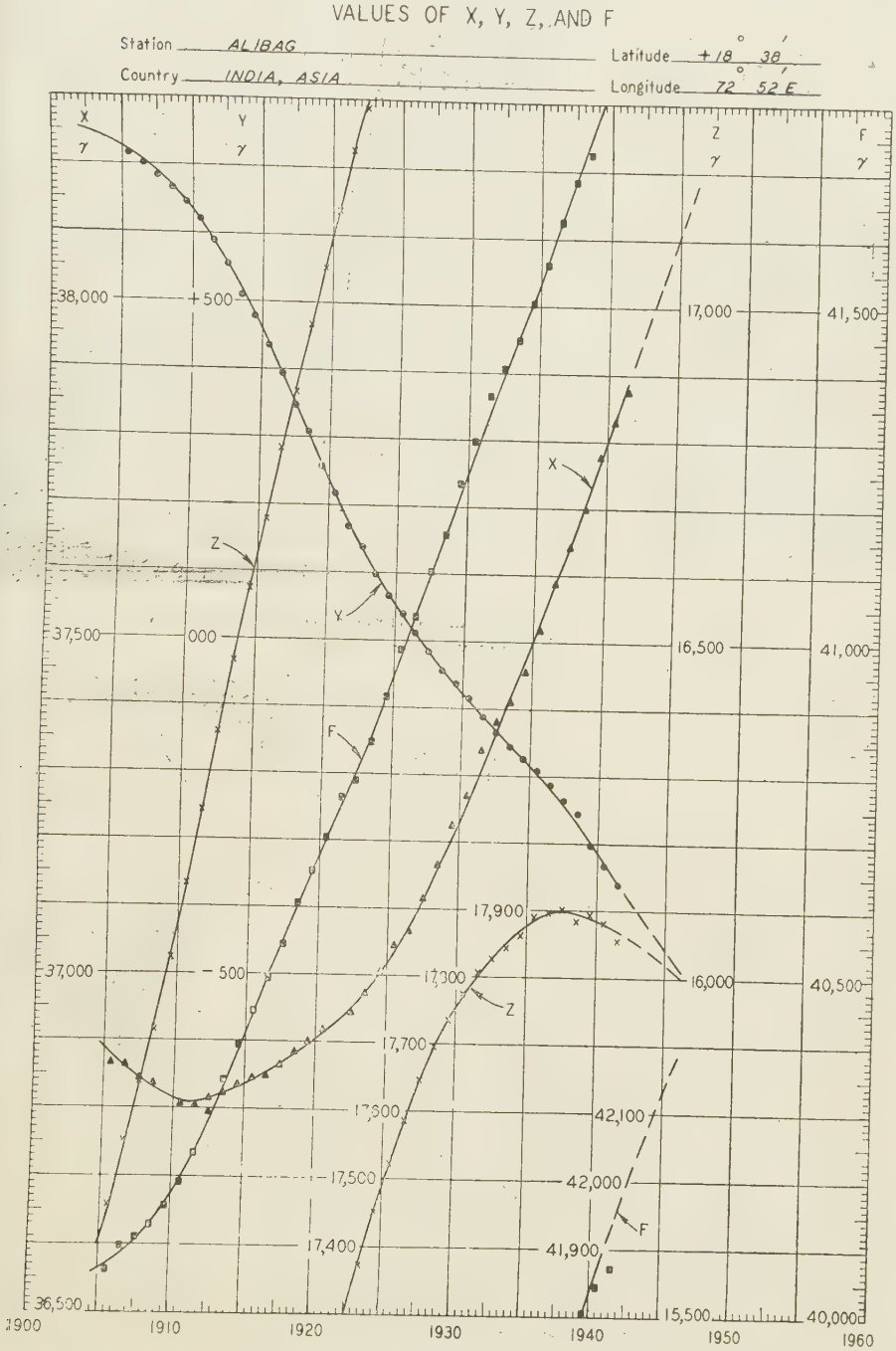

Figure 14 *b*. Specimen graphs for values of *X*, *Y*, *Z* and *F* from 1905 for Alibag.

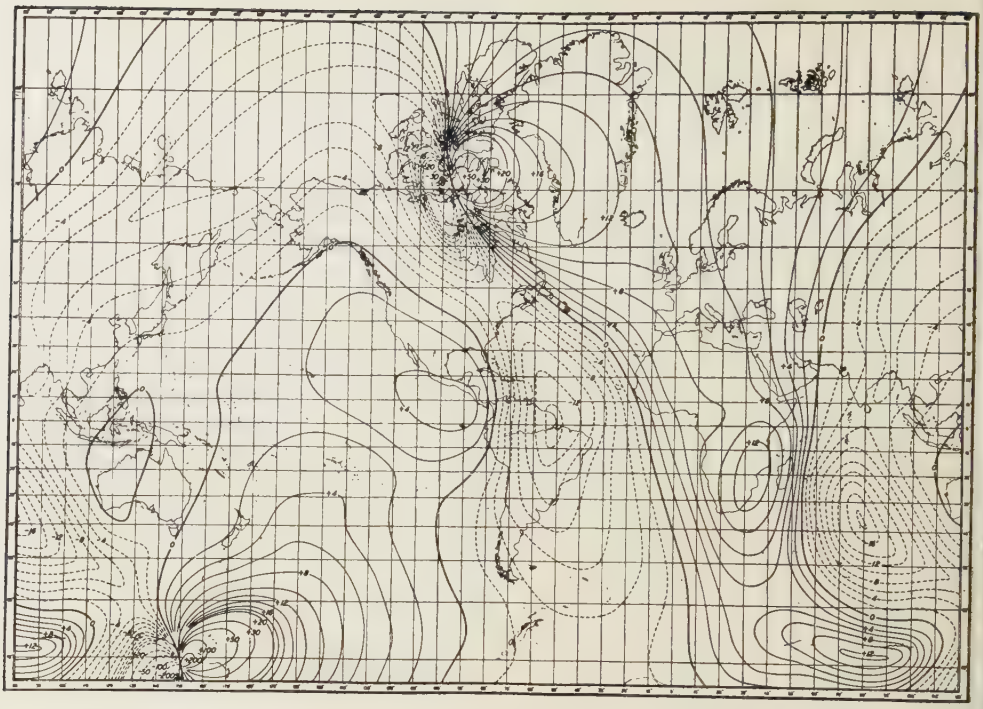

Figure 15 *a*. Isoporic chart for declination (lines of equal annual change in minutes of arc per year), epoch 1912·5.

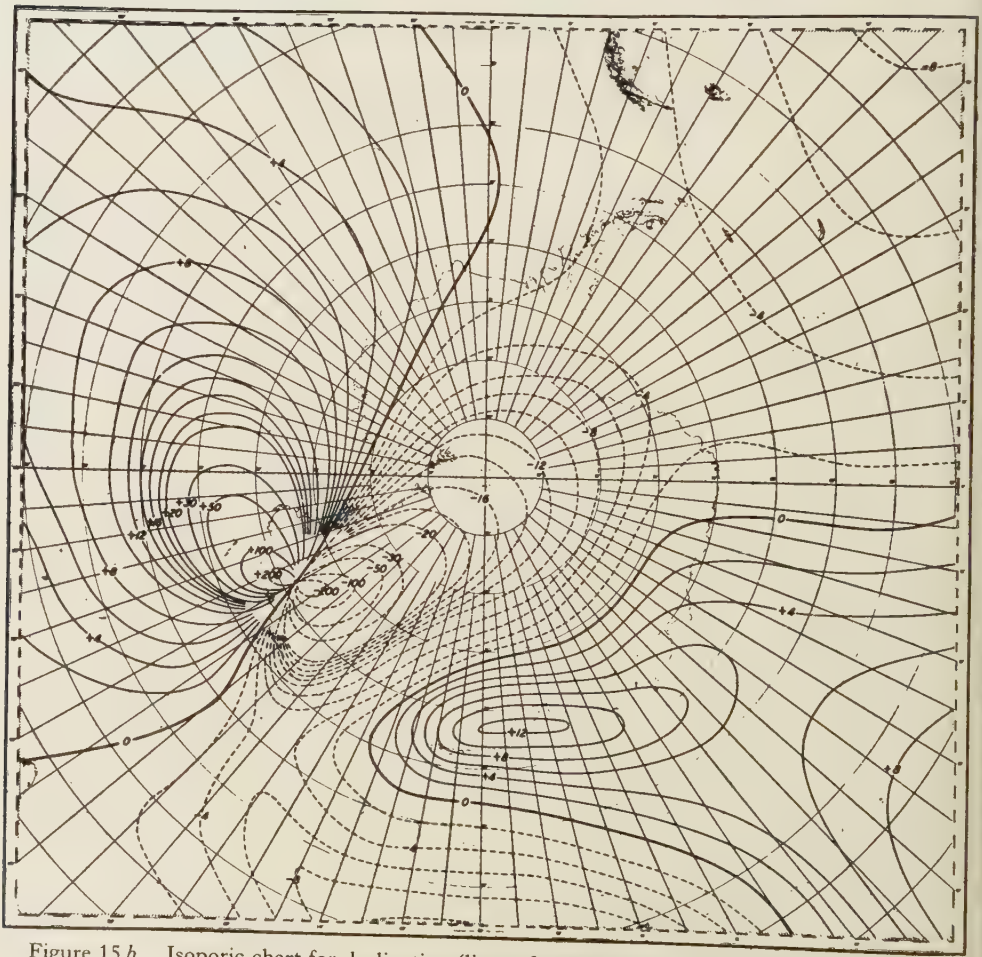

Figure 15 *b*. Isoporic chart for declination (lines of equal annual change in minutes of arc per year), North Polar region, epoch 1912·5.

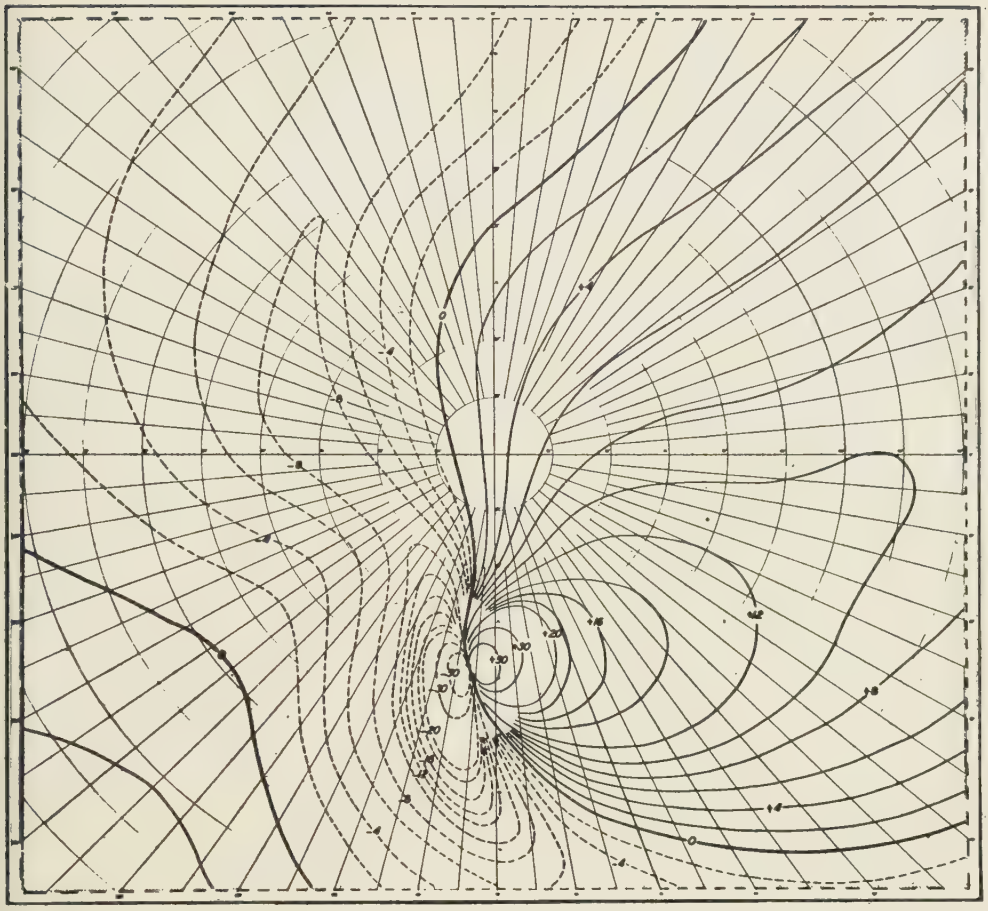

Figure 15 *c*. Isoporic chart for declination (lines of equal annual change in minutes of arc per year), South Polar region, epoch 1912·5.

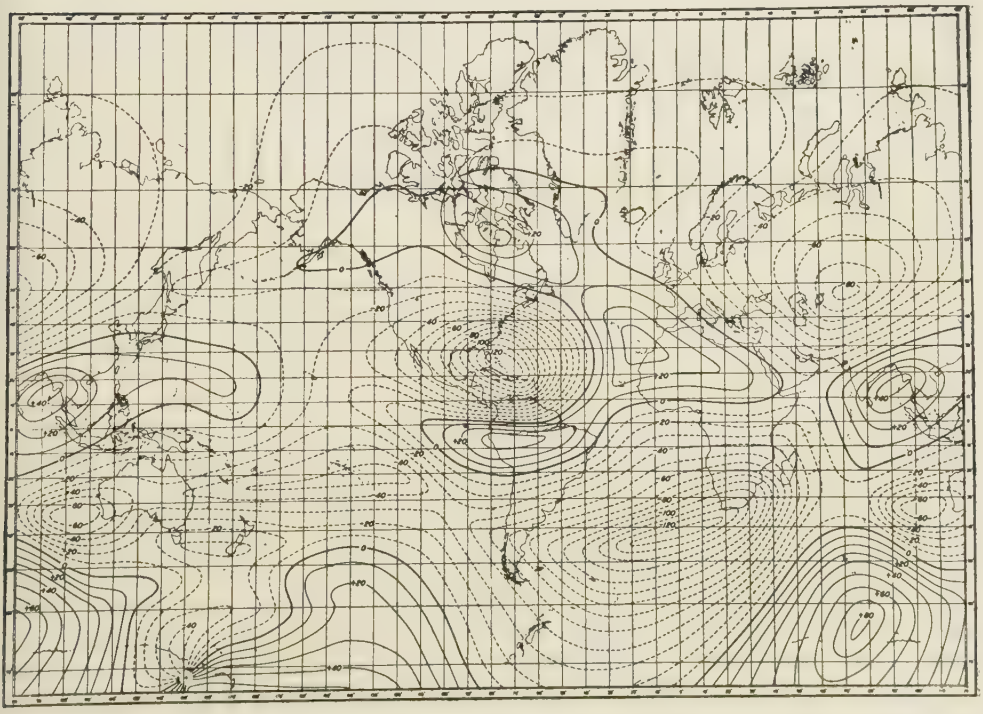

Figure 16 *a*. Isoporic chart for horizontal intensity (lines of equal annual change in gammas per year), epoch 1912·5.

intensity for epoch 1912·5. These were first drawn independently of one another and then adjusted to mutual consistency such that the vertical component of the curl of the field was nearly zero at the corner of each of the 10°-tessera. The adjustments were further continued to ensure vanishing of line-integrals of the northward component along meridians and of the eastward component along parallels of latitude.

The isopors near the magnetic poles are based on a theoretical fit of available data, based on a power-series expansion for the potential due to Chapman (1942). The theoretical fit of secular changes near the dip-poles was also used to estimate

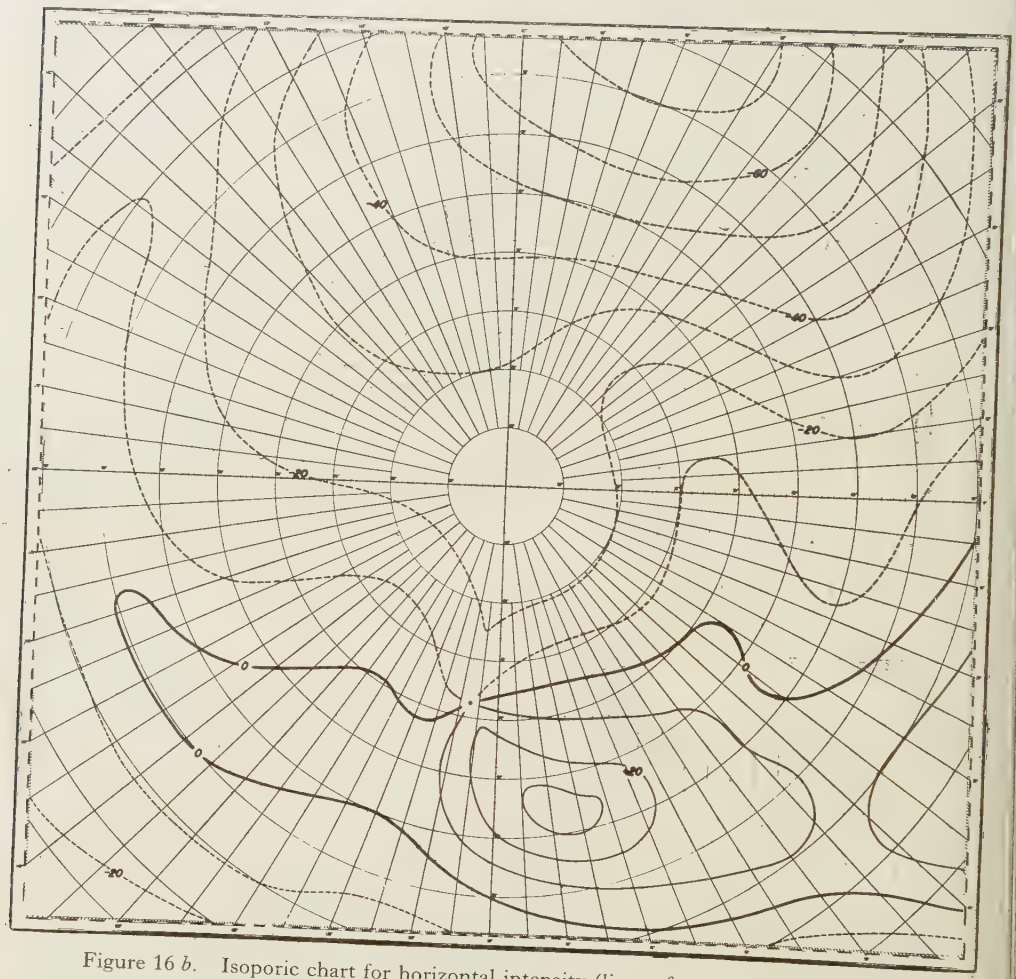

Figure 16 *b*. Isoporic chart for horizontal intensity (lines of equal annual change in gammas per year), North Polar region, epoch 1912·5.

the positions of the north and south magnetic poles given in table 1. These tentative estimates assume the revised location of the north magnetic pole by Wasserfall (1939) based on Amundsen's observations of 1904·5 (Graarud and Russeltvedt, 1925) and of the recent flight of the *Aries*. The spacing between successive positions of the pole at the various intervals of time are adjusted in proportion to the velocities of the pole estimated from secular change. The position of the south magnetic pole is taken as that recently supplied by Farr (1944), the secular change in horizontal intensity and declination, together with the elliptical configuration of the isoclinics, as found by a survey under Mawson in 1912 (Webb, 1925).

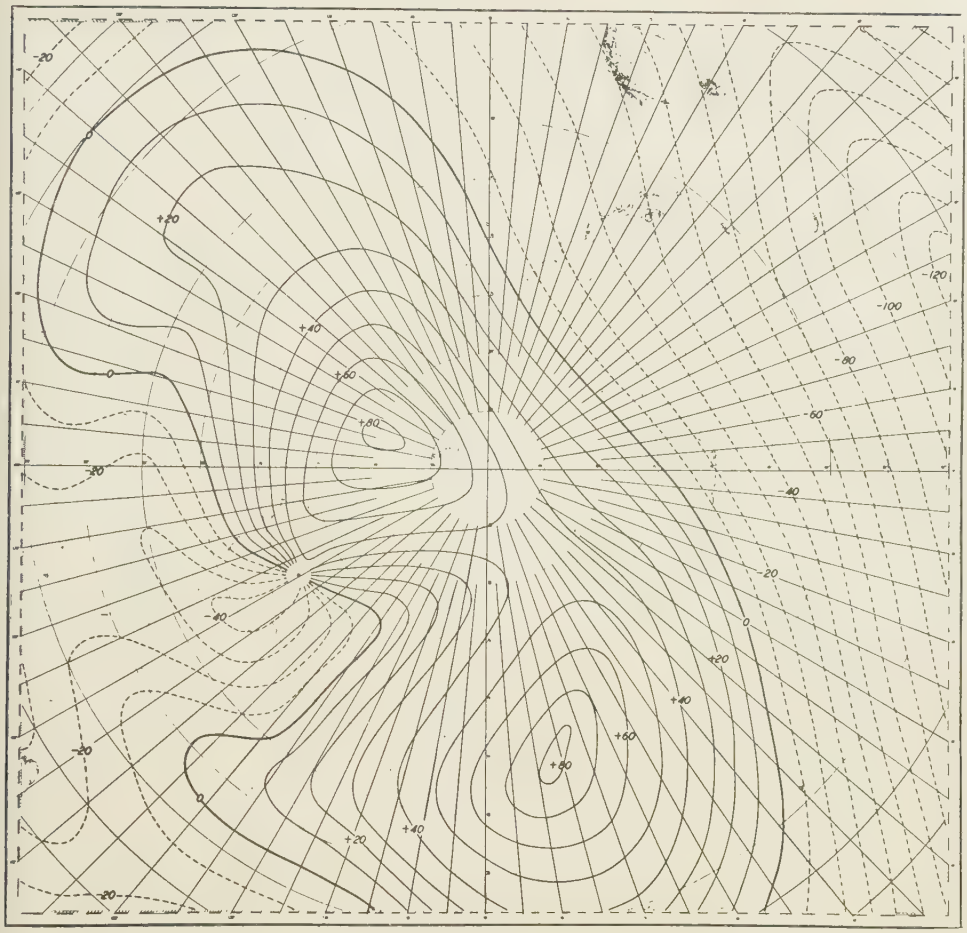

Figure 16 *c*. Isoporic chart for horizontal intensity (lines of equal annual change in gammas per year), South Polar region, epoch 1912·5.

Table 1. Coordinates of north and south magnetic poles.

Date or epoch	Based on secular-change charts for four epochs and positions observed in 1904·5 for north pole and 1912·5 for south pole				Based on isomagnetic charts for four epochs and positions observed in 1904·5 and 1945	
	North latitude	East longitude	South latitude	East longitude	North latitude	East longitude
	°	°	°	°	°	°
1904·5	70·5*	263·5*	....	....	70·5*	263·5*
1912·5	70·9	263·2	71·2†	150·5†	71·3	262·7
1922·5	71·4	262·8	70·2	149·2	72·6	261·8
1932·5	71·9	262·4	69·0	148·1	74·0	260·0
1942·5	72·6	262·1	68·3	146·2	75·4	258·2
1945·0	72·8	262·0	68·2	145·4	76·0‡	258·0‡

* After Wasserfall (1939), based on observations by Amundsen.
† After Farr (1944), based on observations by Mawson.
‡ Preliminary values communicated verbally as observed on *Aries*.

The isoporic charts show agreement with Chapman's rule that the number of nodes equals the number of foci minus 2 (Chapman, 1942). For declination there are eight nodes and ten foci with no ray-poles. For horizontal intensity there are ten nodes, ten foci, and two ray-poles, the latter counting as foci in the application of the rule. New features also depicted for the first time are the distribution of lines near the magnetic poles.

The ten-year successive positions of positive and negative foci in declination and horizontal intensity for the four epochs ten years apart are plotted in figure 17. The most notable feature is the large and systematic departures in position per ten-year period. Tables 2 and 3 list the locations of foci, their magnitudes, and

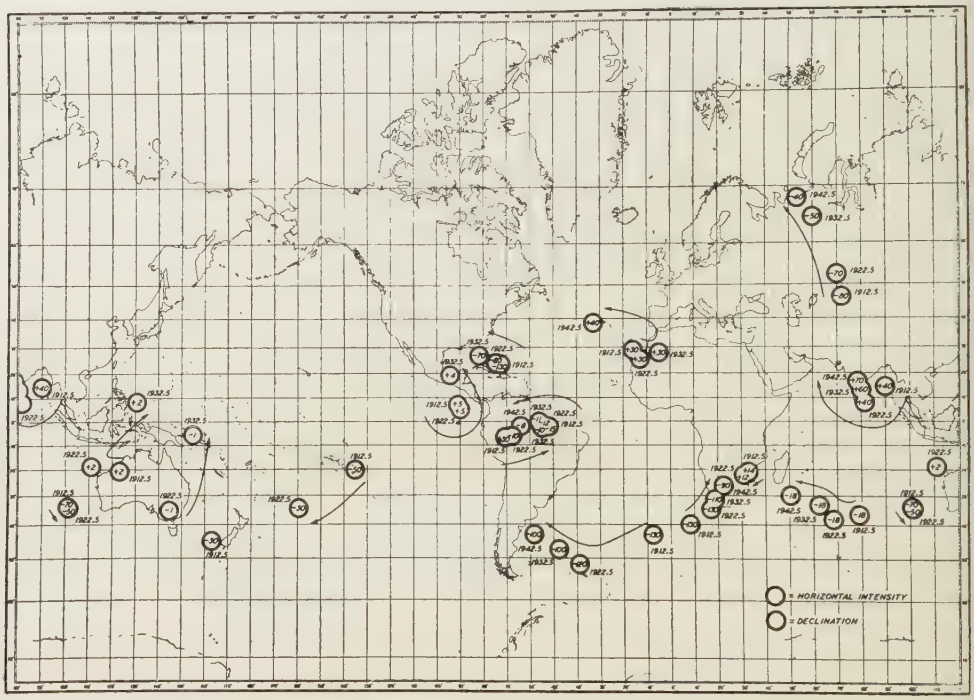

Figure 17. Successive approximate ten-yearly positions and magnitudes of isoporic foci; declination in minutes of arc and horizontal intensity in gammas ; epochs 1912·5, 1922·5; 1932·5 and 1942·5.

positions. In the Eastern Hemisphere the foci of the secular change in horizontal intensity show in general circulation in position to the north and to the west. This finding is in agreement with the general results for the position of the magnetic equator indicated in figure 3. In the Western Hemisphere there seems some indication of motion to the south. The foci for secular change in declination, on the other hand, seem to show motion in earlier years which is in certain cases towards the east, there being less regularity than that noted for the secular change in horizontal intensity.

These new isoporic charts offer several new and interesting possibilities. Although constructed independently of one another, the differences of the chart, epoch by epoch, reveal surprisingly simple consistent patterns. These new patterns represent the time-rate of change of secular variation per ten-year

Epoch											
1912·5			1922·5			1932·5			1942·5		
Latitude	Longitude, E	f	Latitude	Longitude, E	f	Latitude	Longitude, E	f	Latitude	Longitude, E	f
°	°	′	°	°	′	°	°	′	°	°	′
−21	33	+14	−23	31	+12	....	....	....	....	....	....
−36	79	−18	−38	68	−18	−33	62	−18	−30	50	−18
−21	123	+ 2	−19	111	+ 2	+ 7	130	+ 2	....	....	....
....	....	....	−35	144	− 1	− 5	155	− 1	....	....	....
+ 7	269	+ 5	+ 5	270	+ 5	+19	265	+ 4	....	....	....
− 2	308	−15	− 2	306	−12	0	304	−11	− 2	296	− 8

Table 3. Positions and values of maximum and minimum isoporic foci (f) in horizontal intensity for four epochs

Epoch											
1912·5			1922·5			1932·5			1942·5		
Latitude	Longitude, E	f	Latitude	Longitude, E	f	Latitude	Longitude, E	f	Latitude	Longitude, E	f
°	°	γ	°	°	γ	°	°	γ	°	°	γ
+47	72	− 80	+53	70	− 70	+65	60	− 50	60	53	− 40
+15	90	+ 40	+ 8	81	+ 40	+13	80	+ 60	+17	79	+ 70
−34	101	− 70	−35	101	− 50	....	....	....	....	....	....
−20	225	− 50	−35	200	− 30	....	....	....	....	....	....
+24	287	−130	+24	285	− 80	+27	278	− 70	....	....	....
−43	352	−130	−52	321	−120	−47	112	−100	−43	301	−100
−40	8	−130	−35	17	−130	−31	19	−110	−26	22	− 90
− 7	289	+ 30	− 6	292	− 10*	− 3	305	− 10*	....	....	....
+29	344	+ 30	+26	347	+ 30	+28	355	+ 30	+38	328	+ 40
−45	163	− 30	....	....	....	....	....	....	....	....	....

* Cuspidal point.

interval. It should prove interesting to try and relate these new values to other geophysical material. The consistency of the differences found are encouraging in another way. They may indicate that the isoporic charts of the northward and eastward components may be successfully subjected, for the first time, to spherical harmonic analysis. It is planned to undertake such a trial analysis, a check being provided by comparing the separate results for the two components. If the analysis is satisfactory, necessary and desirable improvements in the isoporic charts for vertical component can be made, for instance, in polar regions and over the oceans. The extrapolation of the secular-change field or its equivalent in terms of a thin current-distribution at various depths within the Earth is also contemplated.

An interesting hint of the results of this extrapolation can perhaps be derived by noting the considerable complexity of the isoporic charts in horizontal intensity and declination (no attention should be paid to complexities near the magnetic poles). For this purpose we should preferably have the vector combination of the values of secular change in declination and horizontal intensity. The complexity in pattern must increase rapidly downwards in the Earth, and it is highly probable therefore that this extrapolation would attain unreasonable complexity at modest depths, only a moderately small fraction of the distance to the Earth's centre. We thus restrict the probable site of the origin of secular change to smaller spacial dimensions than hitherto, perhaps within the mantle of the crust of the Earth.

§6. FUTURE GEOMAGNETIC SURVEYS

The present slow and costly methods of geomagnetic surveys seem likely to be supplemented by new techniques and methods, the possibilities of which are now becoming gradually apparent. Clearly, the old procedure of measuring the Earth's field at different times in different places in one region during one year or years and in another at a different epoch is inadequate to all the needs. What is required is a description of the geomagnetic field during a given year based on measurements made during that year at an orderly set of points sufficiently close together and spaced in a manner nearly independent of topography, areas of land and sea, and climate. The prophecy of Chapman in the first Charles Chree Address (1941) that such measurements might well be made from planes promises early realization. A project of this kind is now feasible instrumentally and affords attractive post-war possibilities in application.

The available data are already of necessity squeezed rather hard to obtain useful isomagnetic charts for the epoch 1945 in some regions.

As stated above, charts of first accuracy could not be prepared without extensive and uncertain extrapolations for secular variations; it was impossible to meet military specifications for accuracy on charts on which our magnetic-mine warfare and degaussing were to be based. Mathematical adjustments were resorted to in making up for the paucity of observations. Believing that a non-potential field does not exist, maps were mutually adjusted so that the curl of the field vanished. It is probable that these adjustments give a more accurate representation of the field than could otherwise be obtained, but no amount of mathematical adjustments can ever compensate for absence of observations.

We have sought to improve our knowledge by increasing the accuracy of field

observations, and this step has been important in accurately determining secular changes. However accurately the magnetic elements are determined at a given point, absence of observations at intervening points leaves our information still inadequate for surface isomagnetic maps unless we can establish almost an infinite number of stations.

This condition may now be remedied by magnetic observations with instruments airborne above the surface of the Earth, and thus farther away from magnetic anomalies which depreciate the accuracy of surface observations. Travelling at speeds which a few decades ago seemed fantastic, continuous observations may now be obtained over a large area in a single day, and in a relatively short time a survey of the normal geomagnetic field can be made. The accuracy of such techniques, thanks to the achievements during the past war, can be kept more than adequate for both practical and theoretical purposes. Electronic circuits, monitored by standard cells, can maintain standard electric currents in flight, so that magnetic profiles, accurate to a few tenths of gammas or better, in absolute value in any component can be obtained. The following limitations in the present airborne gear must be overcome:—(*a*) Stable operation must be provided over extended periods of time of measurements of intensity: this is synonymous with negligible base-line and scale drifts in the measuring circuits. (*b*) Modifications are necessary to permit measurement of the angle of declination, that is, the angle between the horizontal projection of the detector coil at any instant and true north or the true heading of the aircraft. (*c*) Observations of the angle of dip will involve the construction of a suitable gyro mechanism to establish an accurate horizontal plane; among problems in this connection are the elimination of centrifugal pseudo-gravity effects due to plane manoeuvres or bumpy air.

The advantages of an effective airborne surveying instrument are great, and include:—(*a*) Ability to cover large areas quickly and continuously. (*b*) Elimination of purely surface anomalies, which are of no significance in the preparation of charts of the normal field; thus, by flying at altitudes of, say, 20 000 feet, smoothed values of the geomagnetic elements are obtained directly, without the necessity of mathematical treatment or judgment on the part of the person processing the data of many agencies and of various degrees of accuracy. (*c*) Ability to define uniquely the magnetic field by flights in two or more horizontal planes.

Perfection of a suitable airborne instrument for magnetic surveying will require both considerable time and funds to overcome these limitations. Nevertheless, the end results possible would justify such expenditure of both time and funds.

Azimuths for declination can be obtained by automatic or manual observations of the Sun, smoothed by gyroscopic stabilization. An example of what can be done with airborne methods is supplied by the recent flights (August 1945) of the *Aries* under sponsorship of the Imperial School of Navigation on the initiative of the Astronomer Royal. In two flights of less than 24 hours over entirely different regions, a new position of the magnetic pole was determined, probably more accurately than ever before. This achievement may be compared with the tedious and dangerous expeditions of Ross and Amundsen to accomplish the same purpose many years ago. The instruments used were magnetic compasses of various types, including gyrostabilized compasses, and also a three-element, saturated-core type of magnetometer. The last, of a relatively low order of accuracy, was not designed for the flights; it had to be held level manually during

an observation. Furthermore, only one element could be observed at a time, and there was no means for continuously recording—thus truly average values of the observations obtained seem consistent. This pioneer work demonstrates what may be expected when magneticians take full advantage of the technical progress of the last five years.

§7. CONCLUSION

During the war, as by-products of military demands, many improved techniques, applicable not only for surveys and experiments but also for mass reductions, analyses, and correlations by machine methods, have been developed. Peace-time applications of these must serve both continuation of surface and initiation of aerial magnetic surveys by our several Governments, as well as interpretative investigations of the vast existing stock of data and that potentially possible in much greater degree in the near future. It is to be hoped that a long period of amity and cordial relations between all nations is now on the horizon. May it give firm foundation in forwarding and coordinating large activities in Earth physics based on that truly scientific principle of entire lack of desire for personal or national aggrandizement.

Finally, such progress requires the combined efforts of the physicist, the mathematician, the geologist, the astrophysicist, and the engineer to enable the geophysicist to solve the riddles of the Earth's main magnetic field and of its secular variations.

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DR. JOHN A. FLEMING,
Third (1945) Charles Chree Medallist.

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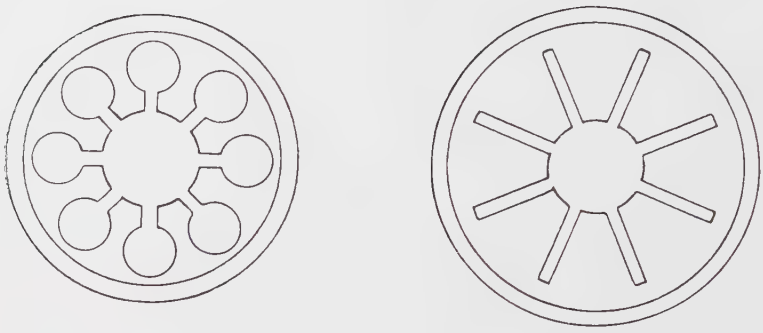

Figure 1. Cavity magnetron resonator systems : sectional view.

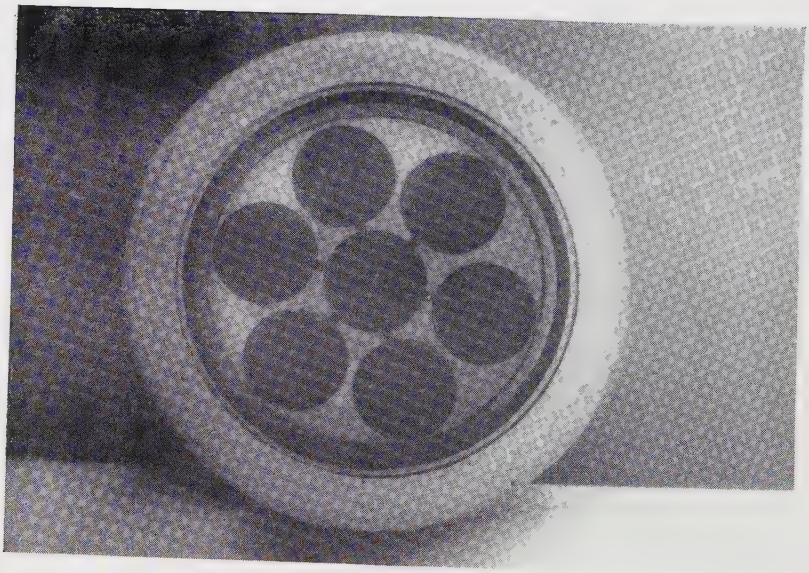

Figure 2. Anode block of one of the earliest cavity magnetrons made in the University of Birmingham.

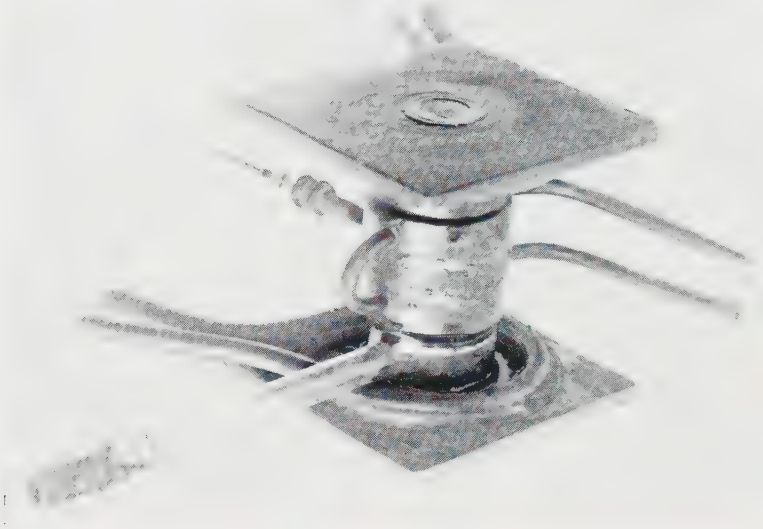

Figure 3. The original cavity magnetron detached from pumping system.

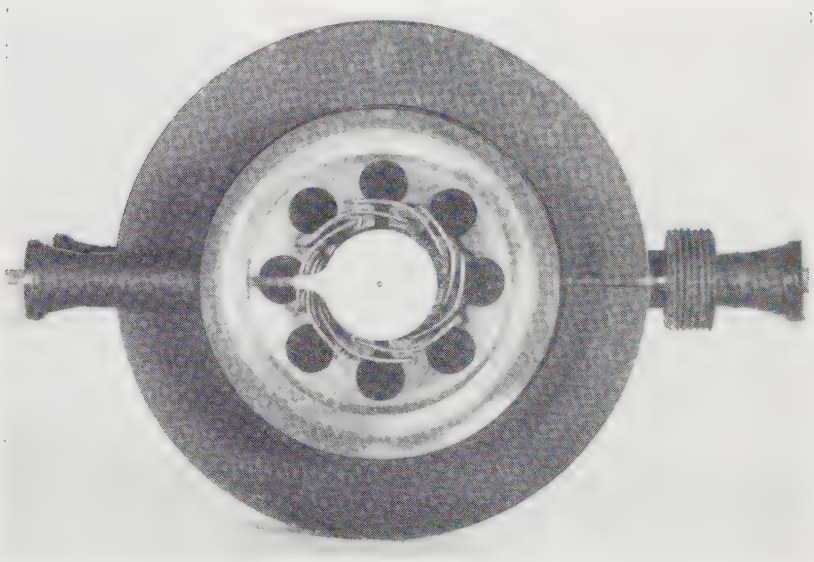

Figure 4. The CV 64, a 9–10-cm. echelon-strapped magnetron much used by the Services. Peak power output ~35 kw.

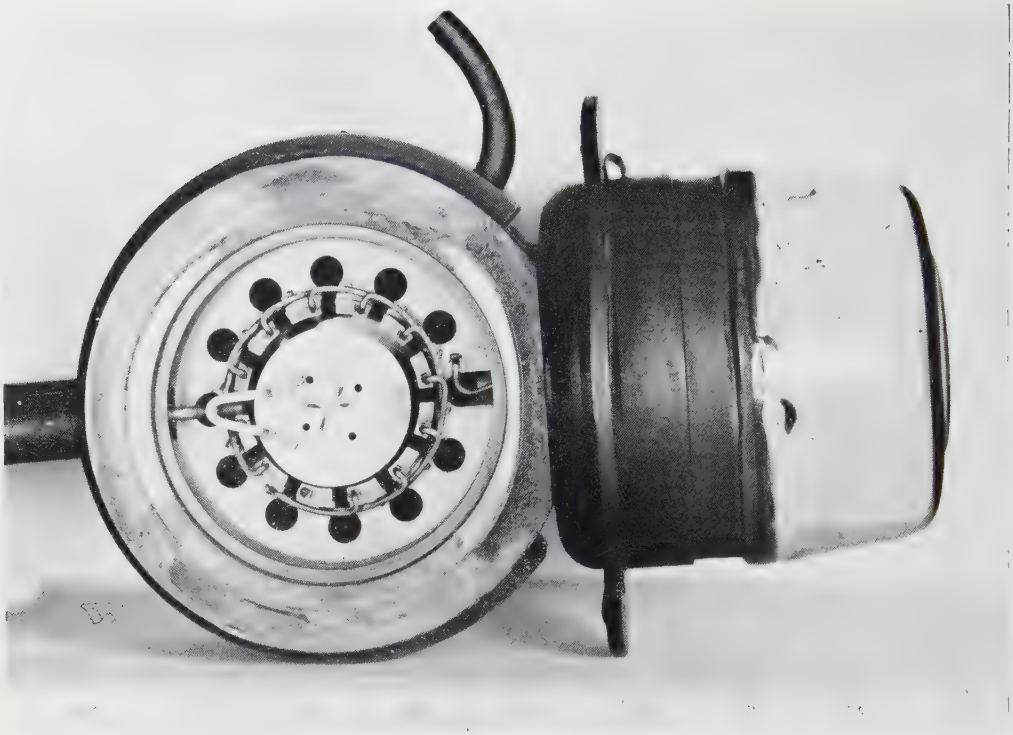

Figure 5. Megawatt magnetron with end-plate removed to show strap construction. (Scale in cm.)

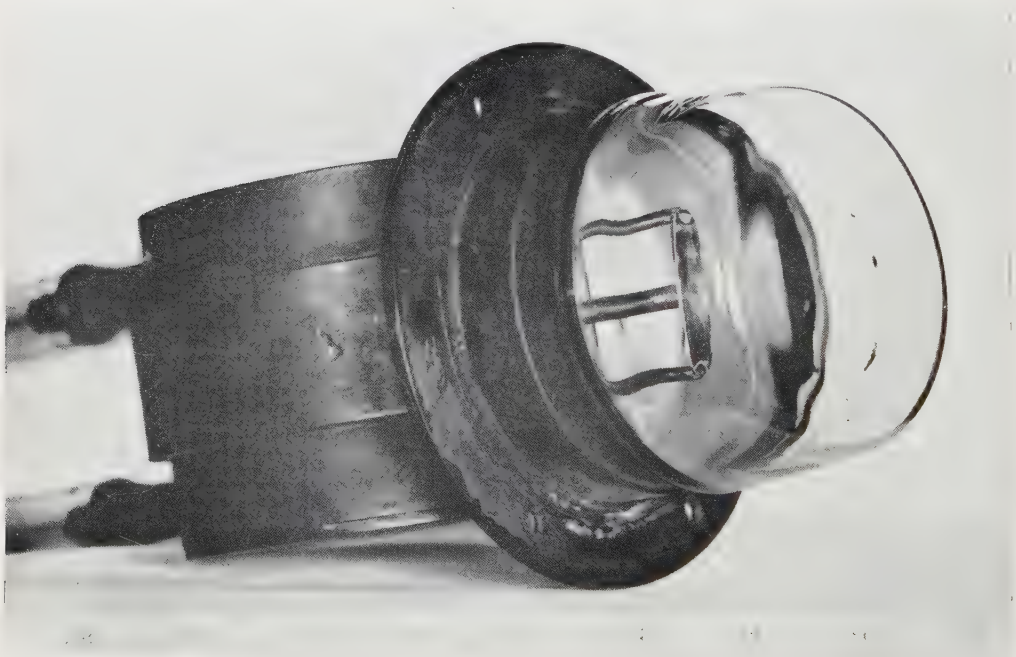

Figure 6. Megawatt megnetron; second view, showing wave-guide output system. (Scale in cm.)

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THE CAVITY MAGNETRON*

By J. T. RANDALL, F.R.S.,
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An Address delivered 12 *December* 1945, *with demonstrations, on the occasion of the presentation of the* 22*nd Duddell Medal to Prof. Randall*

MS. received 17 *February* 1946

In accepting the great honour which you and your Council have bestowed on me I wish to offer my most grateful thanks. I have very little to guide me by way of precedent on this occasion. Either by accident or design the remarks of previous recipients of the Medal have but rarely been reproduced *in extenso*. I must therefore chart my own course. Before I refer to the work you have singled out for special mention I would like to say this:

Not only do I owe a great deal to the laboratories in which I have worked and to their Directors, but I have been specially fortunate in the young people who have collaborated with me. If I had not been trained in Manchester under W. L. Bragg, I might not have developed the interest in crystals which was so valuable as an approach to the problems of luminescence. If I had not spent some years in the Wembley laboratories my approach to valve problems would have been inadequate when I came to think of short-wave generators. And, indeed, if I had not been in the University of Birmingham when Oliphant started his great drive on centimetre-wave work, I should probably have had nothing at all to do with such instruments. I say these things because I believe that at any given time one's past experience affects one's approach to a problem, and determines more than many people think the way in which a problem is solved.

The luminescence work, which started a good deal earlier than that on high-frequency generators, was of two kinds: technical and fundamental. I might mention on the technical side the discovery, by McKeag and myself (1936), of zinc beryllium silicate activated by manganese as a phosphor suitable for excitation by a low-pressure mercury discharge. This phosphor is now a component in many of the mixtures used in the low-voltage fluorescent lamps (see Jenkins, 1945). I think, also, we were among the first to elucidate the technical processes required to obtain a number of high-quality silicate and phosphate phosphors.

On the fundamental side I should like to mention the work of Randall and Wilkins (1946) on the mechanism of afterglow or phosphorescence in different

* This short title has been chosen since it covers the greater part of the subject-matter of the Address, although some reference is made to work on the luminescence of solids.

types of material. In a series of recent papers it is shown how the characteristic thermoluminescence is related to electron traps in the solid; further, that the temperature at which light is emitted is a measure of the depth of the trap below the conduction band in the solid. From a knowledge of the relative numbers of electrons in traps of different depths it was possible to calculate the type of decay law obeyed over a narrow temperature range. Wilkins and Garlick (1945) have followed this work with a paper in which many earlier misconceptions concerning bimolecular decay laws have been cleared away. There is one important problem in this field which someone may wish to investigate: the nature of electron traps in particular cases. No direct attack is possible for obvious reasons. The detailed study of phosphors with more than one activator—rare-earth ions would probably be suitable—will probably give information, not only with regard to the luminescence centre, but about the trap, and optical and thermal excitation energies.

The work in which I suspect you are more interested, however, is the discovery by Dr. Boot and myself of the resonator magnetron in the winter of 1939–40. Later, in 1941, Dr. Sayers made an invaluable fundamental contribution. As the magnetron was immediately successful as a short-wave generator it was taken up for the Government by the large electrical firms, and I should like to pay tribute to the work which was carried out by the staffs of the B.T.-H. and the G.E.C. in this connexion.

As the story has not really been told of how this work arose I should like to spend a little time on the origins of the magnetron as we now know it, and to follow this with a brief reference to later experimental developments. At the close of my talk I hope to show a powerful magnetron in operation, and for generous help in this I wish to thank the Admiralty Signal Establishment.

In the early autumn of 1939, with others whom Professor Oliphant had collected together for this purpose, I began to spend some time on problems connected with centimetre waves, as many other physicists were doing elsewhere. The chief problem was to produce a high-power centimetre wave oscillator. The centimetric nature of the wave was required for direction of beam, and the power for long range. At first Boot and myself spent a few weeks studying the Barkhausen-Kurz tube as a detector device; the greater number of workers in the laboratory, however, were concerned with the klystron (Varian and Varian, 1939), both as an oscillator and as an amplifier, and we were naturally interested in the whole field. The klystron was outstandingly important in that it used for the first time closed, or essentially closed, resonators, fitted with grids through which an electron beam passed, for the production of high-frequency power. Such resonators had been considered by Rayleigh in 1897, and the papers of Hansen (1938, 1939) showed how enclosures of this kind lend themselves to certain modes of electrical vibrations. On any given mode the frequency is largely, if not entirely, dependent on the dimensions of the resonator. When such resonators are constructed of copper, three important features are evident:—(i) low h.f. losses; (ii) wave-length stability; (iii) potential capability of large heat dissipation.

To be set against this, however, is the serious difficulty of getting sufficient power into the electron beam, which is necessarily of small cross-sectional area. We began, therefore, to think of other possibilities. Was there any means of using the desirable features of cavity resonators, and at the same time avoiding the limitations of the electron beam ?

Conventional designs of magnetrons were equally notable in failing to meet the desirable features, and by November 1939 we had decided to try to devise a type of magnetron which did so. The Hansen papers already referred to were by this time available in the laboratory, but it was obvious that the types of resonator considered—hollow spheres, cubes, etc.—could not be associated with the cylindrical anode and cathode of the magnetron. The only other types of short-wave resonator circuit with which we were familiar were Hertz's original loop-wire resonator and a short-circuited quarter-wave line. A three-dimensional version of the first of these gives a cylinder with a slot down one side, and the second becomes a parallel-sided slot $\lambda/4$ deep (see figure 1). Either design was well suited to the arrangement of a number of segments with cylindrical symmetry round a central cathode, and it is also clear that a combined anode and resonator system could be machined from solid copper to allow of good heat transfer and, it was hoped, high-power dissipation.

No theoretical calculations of the wave-length to be expected from a resonator of the cylinder-slot type had been made, but Hertz had shown, and Macdonald (1902) had confirmed theoretically, that the wave-length to be expected from a simple Hertzian dipole was $\lambda = 7{\cdot}94d$, where d is the diameter of the ring.

Six resonators, 1·2 cm. in diameter, with slots $0{\cdot}1 \times 0{\cdot}1$ cm., were employed, and the length of the resonator was 4 cm.; it was hoped that the wave-length of this system symmetrically surrounding an anode-hole of the same diameter would be approximately 10 cm. Preliminary calculations had shown that this design might be expected to operate reasonably well with an applied potential of 16,000 volts and a magnetic field of approximately 1000 oersteds, so that provision of these essentials was a necessary preliminary. In order to save time a D.C. power-supply system was built up, and not a modulated supply, and a large electromagnet was used to give the required field parallel to the cylindrical axis of the valve. Figure 2 shows a view of the first anode block, figure 3 the first resonator magnetron, continuously operated on the pump with glass-metal junctions closed with sealing wax. For various reasons the first trials were delayed, and it was not until 21 February 1940 that the valve was tested. It worked immediately. By various rather crude means it was shown that the output was about 400 W. The wave-length was measured by means of Lecher wires and shown to be 9·8 cm. After this, things moved rapidly, and I will not enumerate the various technical stages of development, which included the making of sealed-off valves by ourselves and by G.E.C., Wembley, incorporating the famous gold-seal device as a means of fixing the end-plates to the valve. We also received great help on the technical side from S. M. Duke, formerly of the Wembley laboratories. Later, Professor Oliphant organized a small production unit for experimental magnetrons, and during its life over a thousand experimental valves were produced.

There have been many stages in the development of the magnetron since 1940, and it is obviously impossible to go into these in detail. It was, of course, important to see to what extent power could be increased. The Wembley and Birmingham laboratories simultaneously introduced large oxide cathodes, although both laboratories were aware that the French had been experimenting on these lines with the older types of split-anode magnetron. During the summer of 1940 we were able to show that the cavity-resonator principle was of general application; any fears that might have existed concerning the applicability to larger numbers of resonators and shorter wave-lengths proved groundless.

During July 1940 experiments on a 5-cm. valve were carried out, and a small amount of power was also obtained at 1·9 cm. using 30 slot resonators. A little later, September 1940, a scaled-down version of the original 10-cm. valve (6 resonators) was operated at 3 cm.

In the early design work we were aided by some elementary considerations. For electrons passing close to the anode without grazing it the operating voltage turned out to be

$$V = 2 \times 10^7 \times \pi^2 D^2 / N^2 \lambda^2 \phi^2 \text{ volts}, \qquad \ldots\ldots (1)$$

where D is the anode-hole diameter, N the number of resonators, and ϕ the phase difference between adjacent resonator tongues.

The corresponding expression for the magnetic field is

$$H = 7{\cdot}24 \times 10^4 / N\lambda \text{ oersteds}. \qquad \ldots\ldots (2)$$

These expressions were undoubtedly based on quite inadequate theory; but as they were found extremely useful in the design of new valves they deserve a small place in the record of developments. It was later shown experimentally at Birmingham that the resonator magnetron will operate with higher efficiency at higher magnetic fields and voltages than those derivable from equations (1) and (2). The first magnetron had an efficiency in the neighbourhood of 10%, and the pulsed power output of some of the early valves was approximately 50 kw. Under the new operating conditions efficiencies of more than twice the above figure were found. Later (1942) it became clear that the high operating conditions were simply an approach to the π-mode.

The mention of power and operating conditions brings up again a feature with which we were concerned in the earliest stages—that of high current or peak-current. From oxide-coated cathodes it is possible to get peak-currents of the order of 5 amp./cm.^{2} of cathode surface, and from this fact it is seen that the limit of moderate beam current which obtains in the klystron is overcome.

Any discussion of cathodes would be incomplete without mention of secondary emission. From various pieces of evidence it became clear that electrons striking the cathode with finite velocity on completion of an orbit produced secondary electrons. This led to an interesting experiment in which a magnetron was made with a simple aluminium cathode entirely unheated. A small quantity of hydrogen was admitted to the valve from a palladium tube and was sufficient to start the build-up of current, and such a magnetron worked normally apart from eventual clean-up of the gas. This experiment showed for the first time the possibility of cold or even water-cooled cathode magnetrons, and foretold the ultimate, but still unrealized, development of such valves for c.w. purposes.

The next stage of development in the magnetron has come to be known as "strapping". In this development I have no personal share, but the story of the magnetron would be grossly inadequate without it. Essentially this is the work of Dr. Sayers at Birmingham.

So far we have considered the magnetron as radiating only one frequency, and, indeed, this can be so. The resonator system has an infinite number of resonant modes with wave-lengths extending from some maximum value to lower and still lower values. The symmetry of the system causes the modes to be bunched together into groups of different frequencies, and each group by analogy with ordinary spectroscopy can be called a multiplet. In general each multiplet will contain $N-1$ modes if there are N resonators. Most of the modes are degenerate, i.e. more than one mode is found for a given wave-length. All

resonant wave-lengths but one (if N is even), or all (if N is odd) are degenerate. Thus in a 10-hole magnetron there are five possible wave-lengths in a multiplet $[1+(N/2-1)]$; one of these corresponds to a single mode, and each of the others to two modes, giving a total of $1+2(N/2-1)=N-1$ modes or $1+2\times4=9$. The fundamental fact of the existence of different modes in a magnetron resonator system showed itself in the phenomenon known as mode-jumping. For a given set of operating conditions a valve could suddenly jump from one of the possible modes to another nearby one. This had obvious disadvantages, and I should like to describe briefly how this was overcome by Sayers at Birmingham.

By connecting alternate segments together by short copper wires the circuit constants of the resonator system are altered, and the general effect can briefly be described as a greater separation of the modes of the multiplet. In nine- or ten-centimetre magnetrons this enables what is called the π-mode of operation, with a phase difference of π between adjacent segments, to be achieved. Examination of the wave pattern inside a cold block in these circumstances shows that the standing wave pattern is very regular in amplitude. The patterns for other modes are not suppressed but are irregular and often only weakly excited. The CV64, a 9–10-cm. strapped magnetron designed at Birmingham and greatly used by the Services, is illustrated in figure 4. The possibility of producing magnetrons having efficiencies of about 50% caused most of the British magnetron effort to be directed towards the design of strapped valves for particular Service applications at both 10 cm. and 3 cm. For the former wave-length no great difficulties were experienced. At first, considerable difficulties existed at 3 cm., but these cannot be detailed here; they were eventually overcome by adopting the American technique of scaling down a 10-cm. valve.

The maximum peak power now obtainable from cavity magnetrons in the 9-10-cm. region is of the order of a megawatt. Two views of such a valve are shown in figures 5 and 6. This is a great advance on the position existing in 1940, and is an example of what can be done by cooperation between the various organizations (universities, firms, establishments) responsible.

All resonator magnetrons operate above cut-off, that is to say, at a value of magnetic field for which there would be no anode current under steady conditions. There is a space-charge between anode and cathode, and the steady states without any high-frequency field on the anode have been analysed in detail by Hartree and Stoner.

The theoretical approach to the problem of the interaction of the electron stream and the high-frequency wave which travels round the anode is extremely complicated; that many of the problems concerning the operation of cavity magnetrons have been solved is in large part due to the efforts of Hartree, Slater, Stoner and their collaborators. Perhaps the most important early theoretical work was carried out by the Slater group. First, calculations were made utilizing field theory to solve the problem of the possible number of frequencies of a multiple resonator system. Secondly, the significance of the space-charge in the magnetron was realized and investigations based on the assumption of a linear field distribution between anode and cathode were made. From a knowledge of known operating conditions it was found possible to calculate the mode of operation of a given valve.

Later theoretical developments were in large part due to English workers, chiefly Hartree and Stoner. The team working under Hartree considered the problem of magnetron space charge more thoroughly and made calculations

based on a field distribution between anode and cathode which was a good deal nearer reality. This work led to the formulation of the now well-known threshold criterion. It was shown that exact synchronism between the outer edge of the space-charge cloud and the rotating wave was not required, but that operation of the valve, once started, would continue over a range of angular velocities of the cloud. The lower limit of angular velocity depended on the amplitude of oscillation ; it was possible to find a valve for which electrons would reach the anode for vanishingly small segment amplitudes for any practical mode-number n. Thus an anode voltage was determined below which no current could reach the anode for a given magnetic field, and this voltage has been called the threshold voltage. The threshold relation may be expressed in equation form by

$$\frac{V_T^2}{10100a^2} = \frac{2H}{n} \cdot \frac{1-b^2/a^2}{21400} - \frac{1}{n^2},$$

where V is the threshold voltage in kilovolts, and n is the effective n number of the mode.

By the middle of 1942 the calculations, which have been very briefly outlined, had given considerable insight into the operation of valves in the 9-cm. to 12-cm. wave-length region. Large discrepancies, however, still existed between the calculated and observed operating conditions for 3-cm. magnetrons. In May 1942, Sayers and Sixsmith, by phase measurements on 3-cm. unstrapped valves with 12 to 16 segments, showed that such valves were not operating in the π-mode; furthermore, it was shown that the direction of the rotating anode-potential wave was opposite to that of the rotating space-charge cloud. This meant that a lower space-charge angular velocity was required for proper interaction for modes other than the π-mode; the lower space-charge velocities corresponded much more closely with the observed operating conditions. The Hartree group worked ou the required modification to the earlier resonance criterion. Under the new conditions it was shown that for electrons to reach the anode at magnetic fields well above cut-off value, and for vanishingly small high-frequency amplitude a certain degree of synchronism is required between the rotating space charge and one of the spatial components of the rotating electric field in the anode-cathode space.

The theoretical work on the magnetron which has been referred to has not yet been published, and it is extremely difficult to cover it adequately in such a short space. It is to be hoped that those directly concerned will take steps to give a comprehensive account in the near future.

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AN APPARATUS FOR ACCURATE MEASUREMENT OF THE ACOUSTIC IMPEDANCE OF SOUND-ABSORBING MATERIALS

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MS. received 15 *October* 1945

ABSTRACT. This paper describes the principles and the construction of an apparatus for the precise measurement of the acoustic impedance, at normal incidence, of sound-absorbing materials by a stationary-wave method for frequencies in the range 100 to 5000 c/s. From measurements made on a sample of material $1\frac{3}{4}$ inches in diameter, backed by a substantially rigid wall, the magnitudes of the resistive and reactive components of the impedance may be calculated with an accuracy of about one per cent.

The theory of the method is discussed with particular reference to the influence thereon of the attenuation of sound associated with the walls of the tube in which the standing wave is formed, and it is shown that in addition to the correction which must be provided to the elementary expression for the " standing-wave ratio ", the finite attenuation in the tube leads to an additional correction to the expression for the distance from the sample of the first minimum of pressure. Attention is drawn to convenient methods which facilitate calculation of the results, and typical results of measurements of the impedance of a sample of porous sound-absorbing material are shown.

§ 1. INTRODUCTION

In recent years interest has been steadily growing in the recognition of the acoustic impedance, rather than the absorption cofficient, as the chief characteristic of the performance of acoustical materials. The advantages which follow from a knowledge of the impedance are twofold. Firstly, modern theories relating to the absorption of sound both in enclosures (Morse, 1939a) and in ducts (Morse, 1939b) are formulated in terms of the impedance of the walls for normal incidence. Secondly, a knowledge of the separate resistive and reactive components of impedance of an absorbing material simplifies the task of relating the deficiencies in the absorbing power to the physical structure of the material.

Various methods for the measurement of acoustic impedance have been devised, and are referred to in a recent paper by Beranek (1940). There is a paucity of published results by such methods, and until the publication of Beranek's paper, no substantial accumulation of results of adequate accuracy had appeared. Beranek attributes this condition to the difficulties inherent n the methods of measurement previously in use, and proceeds to describe an apparatus of his own design which he considers to be capable of producing results of the required accuracy.

In the present paper, a modification of the apparatus originally used by H. O. Taylor is described. This apparatus is capable of an accuracy of the same order as that of Beranek and possesses, in the author's opinion, advantages of

simplicity in design and in theory of operation. Measurements of acoustic impedance can be made with this apparatus at frequencies in the range 100 to 5000 c/s.

§ 2. THE METHOD AND ITS THEORY OF OPERATION

The principle of the method is the same as that used by Paris, which in turn is based on an extension of that devised by Tuma, and later used by H. O. Taylor for the measurement of the absorption coefficient. The sample of material closes one end of a uniform tube, and plane sound waves are set up in the tube by means of a source of sound at the other end. The stationary-wave pattern which is formed in the tube is explored by means of a microphone, and measurements are made of the ratio of the maximum to the minimum pressure in the system and of the distance of the first minimum from the face of the sample. From these two quantities the two components of the acoustic impedance of the material are obtained.

The derivation of the algebraic expressions which relate the resistive (R) and reactive (X) components of the acoustic impedance of the sample to the ratio of maximum ($p_{\max}$) to minimum ($p_{\min}$) oscillatory pressure in the stationary wave and to the distance (d_N) of the Nth minimum from the face of the sample is given in standard text-books on sound * for the ideal case of negligible dissipation in the stationary wave. Thus it can be shown that the acoustic impedance Z of the sample is given by

$$Z = R + jX = \rho c \tanh(\gamma + j\delta), \qquad \ldots\ldots(1)$$

where ρ is the density and c is the velocity of sound in the air. The quantities γ and δ are obtained from the measured values of $|p_{\max}|/|p_{\min}|$ and of d_N by the relations

$$|p_{\max}|/|p_{\min}| = \coth\gamma, \qquad \ldots\ldots(2)$$

$$d_N/\lambda = \tfrac{1}{2}N - \delta/2\pi, \qquad \ldots\ldots(3)$$

where λ is the wave-length of sound in the tube.

The assumption of negligible attenuation in the tube is not justifiable where precise measurements (i.e. to about 1 % in R and X) are required. The principal effect of finite attenuation in the tube is to cause the maxima and minima to alter progressively in magnitude with distance from the sample. An example of this is shown in figure 1, which gives a graph of oscillatory pressure against distance from the sample for a specimen of rock-wool 1″ deep at 1500 c/s., backed by a massive piston. The theory of this effect has been formulated by Davis and Evans (1930) and by Hall (1939), who show that to a first approximation the maxima can be regarded as substantially unaffected, whilst the minima increase linearly as the distance from the sample increases.

The finite attenuation in the tube leads also to a shift in position of the minima. The existence of this shift must have been clear to Davis and Evans, but its magnitude does not yet appear to have been discussed in detail. Neglect of the

* Morse (1936). Note that Morse describes the time dependence by $\exp(-j\omega t)$, whereas here we use the alternative $\exp(j\omega t)$. The difference results in a corresponding change of sign in the phase angle.

shift can cause appreciable error in precise measurements of impedance, and an outline of the effect will, therefore, be given.

Assume that the acoustic disturbance in the tube may be represented by means of two oppositely and axially directed trains of plane waves of frequency $\omega/2\pi$, wave-length constant $\beta\ (\equiv 2\pi/\lambda)$ and attenuation constant α. The expression for the oscillatory pressure p_x at a section of the tube distance x from the face of the sample is

$$p_x = A \sinh \{\alpha x + \gamma + j(\beta x + \delta)\},$$

whence the amplitude of the oscillatory pressure is given by

$$p_x = B\,[\cosh 2(\alpha x + \gamma) - \cos 2(\beta x + \delta)]^{\frac{1}{2}}, \qquad \ldots\ldots(4)$$

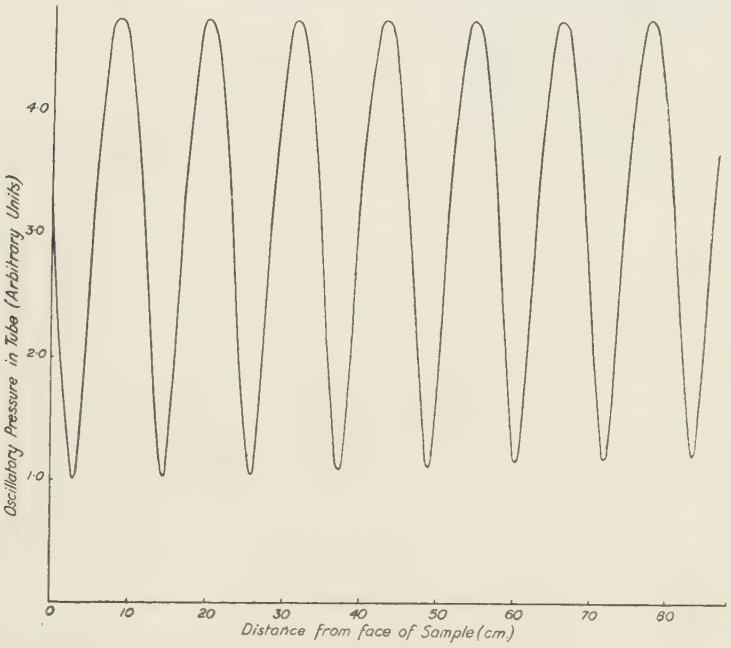


Figure 1. Typical graph of pressure against distance for sample.

where B is a constant independent of x, and where γ and δ are quantities which are associated with the conditions of reflection at the sample, and which satisfy equation (1); thus $e^{-2\gamma}$ is the ratio of the amplitudes of reflected to incident wave at the sample, and 2δ is the phase change accompanying reflection.

Stationary values of p_x occur when

$$\alpha \sinh 2(\alpha x + \gamma) + \beta \sin 2(\beta x + \delta) = 0, \qquad \ldots\ldots(5)$$

i.e.

$$\sin 2(\beta x + \delta) = -\frac{\alpha}{\beta} \sinh 2(\alpha x + \gamma).$$

If α/β is much less than π, the solutions of this equation can be written

$$2(\beta x + \delta) = 2N\pi - \frac{\alpha}{\beta} \sinh 2(\alpha x + \gamma)$$

and $$2(\beta x+\delta)=(2M+1)\pi+\frac{\alpha}{\beta}\sinh 2(\alpha x+\gamma),$$

whence, for p_x a minimum, the distance d_N to the sample is given by

$$d_N=\tfrac{1}{2}N\lambda-\frac{\delta\lambda}{2\pi}-\frac{\alpha}{2\beta^2}\sinh 2(\alpha x+\gamma), \qquad \ldots\ldots(6a)$$

and for p_x a maximum, the distance D_M to the sample is given by

$$D_M=\frac{2M+1}{4}\lambda-\frac{\delta\lambda}{2\pi}+\frac{\alpha}{2\beta^2}\sinh 2(\alpha x+\gamma). \qquad \ldots\ldots(6b)$$

In the measurement of impedance, particular interest centres on the position of the first minimum. Rearranging (6 *a*),

$$d_N/\lambda=\tfrac{1}{2}N-\frac{\delta}{2\pi}-\frac{\alpha}{2\beta^2\lambda}\sinh 2(\alpha x+\gamma). \qquad \ldots\ldots(7)$$

Comparison of (7) with (3) shows that the effect of finite attenuation in the tube is to reduce the distance between the sample and the first minimum by an amount

$$\frac{\alpha}{2\beta^2}\sinh 2(\alpha x+\gamma).$$

The effect of attenuation on the ratio $|p_{\max}|/|p_{\min}|$ may be found by substituting from (6 *a*) and (6 *b*) in the expression for p_x. Thus for the ratio of the Mth maximum to the Nth minimum

$$|p_{\max}|/|p_{\min}|=\left[\frac{2\cosh^2(\alpha D_M+\gamma)-\frac{\alpha^2}{2\beta^2}\sinh^2 2(\alpha D_M+\gamma)}{2\sinh^2(\alpha d_N+\gamma)+\frac{\alpha^2}{2\beta^2}\sinh^2 2(\alpha d_N+\gamma)}\right]^{\frac{1}{2}}.$$

The importance of the effect of attenuation in the tube depends on the magnitude of the attenuation coefficient. For tubes with smooth walls and of diameters greater than six inches, the values of α are very small. Experiments on the apparatus which is to be described later show that for frequencies up to 3000 c/s. the attenuation constant for a tube of diameter 4·45 cm. is given by $1{\cdot}37\times10^{-5}\sqrt{f}$ for room temperatures of 22° c. This figure corresponds to air of low humidity, and some modification may well be expected for damp air (Knudsen, 1933). The above figure is in good agreement with the theoretical attenuation constant deduced from the well-known Helmholtz-Kirchhoff analysis of the passage of sound waves along smooth-walled, circular pipes (viz. $1{\cdot}33\times10^{-5}\sqrt{f}$ at 23° c.).

Where γ is small compared with αd_1, the shift, which is a maximum at low frequency, is less than 0·1 % of d_1. As γ approaches unity, however, the correction becomes

$$\frac{\alpha}{2\beta^2}\sinh 2\gamma,$$

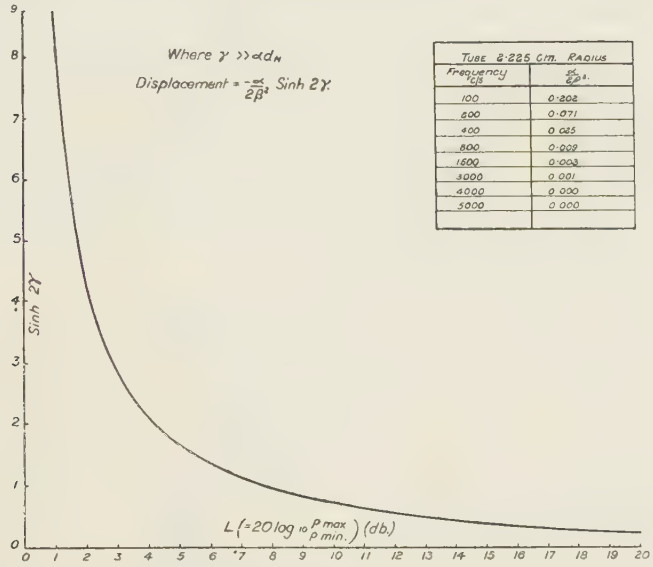
and is appreciable at low frequencies. Figure 2 shows a plot of $\sinh 2\gamma$ against the ratio $|p_{\max}|/|p_{\min}|$ expressed in terms of $L=20\log_{10}|p_{\max}|/|p_{\min}|$, and included on the graph is a table of $\alpha/2\beta^2$ for various frequencies in the range of the apparatus. For a stationary wave with $|p_{\max}|/p_{\min}|=2$, i.e. $L=6{\cdot}0$, $\sinh 2\gamma=1{\cdot}34$ and the

correction to d_N becomes 2·7 mm. at 100 c/s. For a tube of smaller diameter this correction would become more important.

For the values of α and β appropriate to the present apparatus, the ratio of maximum to minimum pressure differs negligibly from

$$\cosh(\alpha D_M+\gamma)/\sinh(\alpha d_N+\gamma).$$

Successive maxima lie on a cosh curve and minima lie on a sinh curve. The maxima vary very little with distance, whilst the minima vary approximately


Where $\gamma \gg \alpha d_N$
Displacement $= \frac{-\alpha}{2\beta^2}$ Sinh 2γ
Sinh 2γ
L (=20 log $_{10}$ $\frac{p\ max}{p\ min.}$) (db.)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
0 1 2 3 4 5 6 7 8 9

Tube 2·225 Cm. Radius	
Frequency c/s	$\frac{\alpha}{2\beta^2}$
100	0·202
200	0·071
400	0·025
800	0·009
1600	0·003
3000	0·001
4000	0·000
5000	0·000

Figure 2. Correction to position of minimum, for effect of attenuation in the tube.

linearly with d_N, except where γ is large, in which case the total variation of minima with distance is correspondingly small. Account may conveniently be taken of the effect of attenuation on the ratio of maximum to minimum by plotting $|p_{\max}|$ and $|p_{\min}|$ against distance from the sample and extrapolating the envelopes to the position of the sample. The ratio $|p_{\max}|/|p_{\min}|$ extrapolated in this fashion then satisfies the equation (2), which is deduced by neglecting attenuation.

§ 3. DESCRIPTION OF APPARATUS

For most practical problems in which sound-absorbing materials are used, it is necessary to know the magnitudes of the components of the acoustic impedance to within 5 %. A better accuracy than 5 % is required where measurements of impedance are to be used in the discussion of the detailed mechanism by which sound is absorbed in the material. It is, therefore, desirable that the measuring apparatus should be capable of providing results with an accuracy of 1 % in both resistive and reactive components for frequencies in the range of 100 to 5000 c/s. At lower frequencies than 100 c/s., available materials have such poor absorption that accurate measurements of impedance are at present of little practical interest.

Where determination of the normal sound-absorption coefficient to within 0·01 is concerned, measurements present no great difficulty, since the ratio

$|p_{max}|/|p_{min}|$ alone is involved. For measurements of impedance a knowledge of the position of the minima is also necessary, and here the requirements are stringent. The impedance of a layer of almost any material used for absorption of sound possesses a large negative reactance term for frequencies low in the acoustic range. The reactance is associated with the finite depth of the absorbing material, and is due to the high elasticity of the air trapped in the layer. Thus $R/\rho c = 1{\cdot}01$, $X/\rho c = -16{\cdot}7$ for a 1″ thick layer of rock-wool at 100 c/s. In this case d_1 is about 83 cm., λ about 345 cm. and d_1 must be measured to within 0·3 mm. in order to give R and X with an accuracy of 1 %. If such precise measurement is to be significant, the following conditions must be satisfied :

(i) the bore of the tube must be uniform and the walls smooth and substantially rigid;

(ii) the sound field in the tube must consist of plane, uniform, axially directed waves;

(iii) the face of the specimen of material must be substantially plane and mounted normal to the axis of the tube;

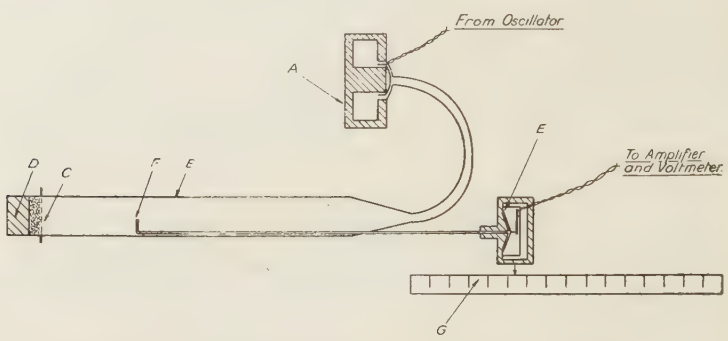


Figure 3. General arrangement of measuring apparatus

(iv) the microphone used for exploration of the sound field must not itself appreciably affect the field and must be sensitive and stable;

(v) the measurement of position of microphone orifice must be accurate to about 0·1 mm.;

(vi) the pressure measured with the microphone must arise only from sound of single frequency.

The apparatus was designed and constructed so as to satisfy these requirements. The conditions under which plane, axially directed waves may be set up in a tube are well known. For a circular tube Rayleigh has shown theoretically that serious departures from uniform plane waves will not occur provided that the diameter of the tube is less than $0{\cdot}58\lambda$, where λ is the wave-length of the sound of highest frequency used in the tube. Experiment shows that in practice the field is substantially uniform until the diameter of the tube reaches nearly twice this value. Thus, taking Rayleigh's limit, if measurements are required at frequencies as high as 5000 c/s., the diameter of the tube should not exceed 4·0 cm. A diagram of the apparatus is shown in figure 3. The tube (B) has an internal diameter of 4·45 cm. and a smooth brass wall $\frac{1}{8}$″ thick. The

maximum variation in cross-sectional area along its length amounts to less than 0·1 %. The specimen of absorbent material (C) is mounted in a detachable specimen tube which is closed at the rear end by a massive (1000 gm.) close-fitting brass piston (D). The positions of the piston and specimen are adjusted so that the front face of the specimen lies accurately in the plane (H) of the flange of the specimen tube. The sound enters the tube through a conical end-piece (J) from a moving-coil loud-speaker unit (A) (Vitavox: type N), which in turn is supplied with electrical power from a valve oscillator of conventional type. The sound field in the tube is explored with a moving probe-microphone (E), the position of which, along the axis of the tube, is measured by means of a cursor sliding over an accurately engraved steel metre scale (Chesterman: No. 1367) (G) divided in millimetres.

The mode of construction of the probe-microphone is important. The microphone consists essentially of a long tube of stainless steel of internal diameter 0·241 cm., external diameter 0·316 cm. and length 104 cm. One end of the tube is connected to a small cavity, of volume about 2 cm^3, in front of the conical diaphragm of an otherwise hermetically sealed rochelle-salt, piezo-electric microphone. The passage of sound waves to the microphone by any path other than along the tube is prevented by the presence of the massive ($\frac{1}{2}''$ thick) shell of the microphone casing, which is made from a steel bar of diameter 3″ bored out so as to take the microphone unit. The other end (F) of the probe-tube is open and communicates with the sound field. The response of the microphone at any one frequency is proportional to the oscillatory pressure at the mouth of the probe-tube, and since the impedance presented by the mouth of the probe-tube is high, the acoustic pressure at the mouth of the probe is effectively that existing at the same position in the sound field when the microphone is withdrawn. The modification of the sound field caused by obstruction due to the probe-tube is negligible, since the area of cross-section of the probe-tube is less than $\frac{1}{2}$ % of that of the main tube. The construction of the apparatus is shown in more detail in figures 4 and 5. Figure 4 is a side view of the trolley (with indicating meter removed) and of the driving end of the tube. Figure 5 is an end view of the trolley, looking from the direction of the tube.

Vibration which exists in the wall of the probe is prevented from reaching the microphone by the interposition of a sleeve (K) of cycle valve-rubber between the two components. Tests were made of the response of the microphone in the normal condition and with the orifice of the probe-tube closed with a short length of brass rod. A typical test at 1000 c/s. showed that the response of the microphone was reduced at least by 50 db. when the orifice was closed. The microphone has proved in practice to be stable and sensitive, and no difficulty is experience in determining the acoustic pressure at maxima or minima in the tube to within 0·1 db. for any frequency in the range from 100 c/s. to 5000 c/s.

The probe-tube passes into the main tube through a long hole bored in the conical end-piece, and for the most part of its length rests on the bottom of the main tube. The probe-tube, rather than the microphone casing, is clamped rigidly to the saddle of the microphone trolley (L), whilst the microphone casing is resiliently supported above the trolley by means of the sponge-rubber (M) and the ring (N). The scale cursor (O) is mounted close to the fixing point

of the probe, and in consequence the scale readings give accurately the successive positions of the probe orifice.

The microphone trolley is mounted on four wheels: two grooved to fit one rail of the angle-brass track (Q in figure 5) and the other two ungrooved. Coarse

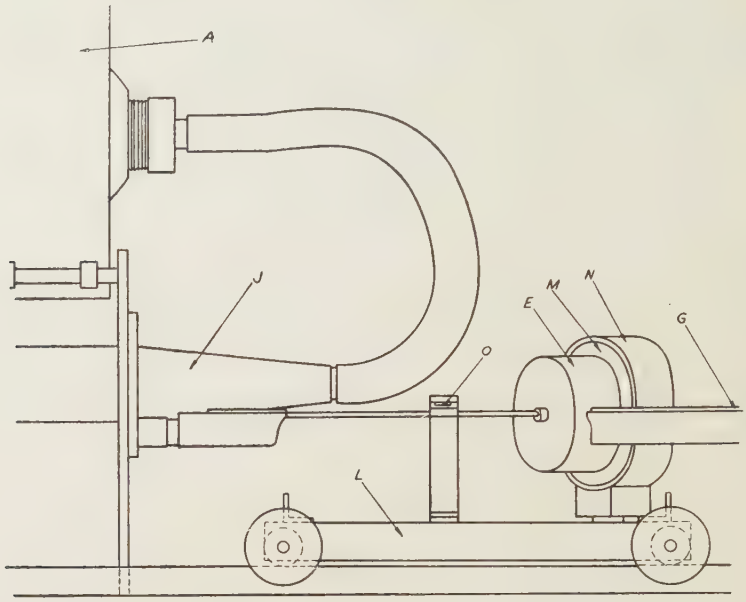


Figure 4. Detail of driving unit and microphone trolley.

adjustment of position of the microphone is made by pushing the trolley bodily along the track, and fine adjustment by means of the knurled knobs (R), which form extensions to the near-side wheels. A four- rather than three-wheeled construction for the trolley was used to provide greater stability of the trolley.

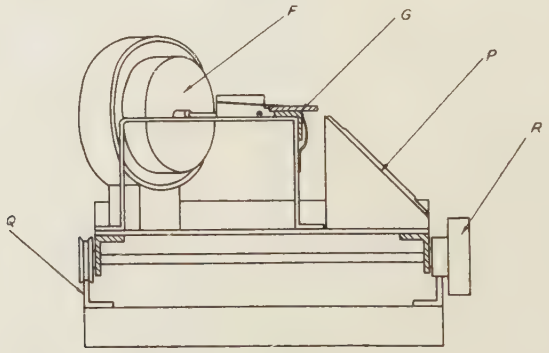


Figure 5. End view of microphone trolley.

The frame of the trolley was made to be sufficiently flexible for each of the wheels to remain in good contact with the rails.

The output voltage from the microphone is amplified by means of a multi-stage valve-amplifier, and is then passed through an electrical analyser, tuned to the working frequency, to an indicating meter (P in figure 5) of the moving-coil,

rectifier-type ($2\frac{1}{2}''$) which, for convenience, is mounted on the trolley. The amplifier effectively excludes such harmonics of the fundamental frequency as may exist in the sound field.

In the plate facing p. 264, (*a*) shows a general view of the apparatus, and (*b*) gives a more detailed view of the driving end of the tube and of the microphone trolley.

Typical operation of the apparatus is as follows :—A sample of acoustical material is cut by means of a circular pastry-cutter so as to fit accurately into the specimen tube. The front face of the material is adjusted so as to lie accurately in the plane of the face of the flange of the specimen tube, and the brass piston (D) is moved so as just to touch the rear face of the specimen. The sound field of appropriate frequency is set up in the tube; there is, of course, no necessity to "tune" the tube since the form of the sound-field, looked at from the "sample" end of the tube, depends only on the nature of the reflection at the sample. The reading of the indicating meter, which is scaled in db., now corresponds to the pressure at the end of the microphone tube. The scale (G) and cursor have previously been set up so as to give a reading of zero when the axis of the upturned end portion of the probe-tube is exactly in the plane of the face of the flange to which the specimen tube attaches. Corresponding readings of indicating meter and of scale give the acoustic pressure in terms of the distance from the face of the sample. The trolley is moved smoothly away from the specimen, and several successive readings are taken of pressure at maxima and minima. From these measurements the ratio at the sample of maximum and minimum pressure is obtained, if necessary by extrapolation of a graph of the ratio, in linear units.

Positions of minima relative to the sample are now found, and are conveniently determined from the mean of two readings, equally spaced about the minimum, each corresponding to the meter pointer passing through an arbitrarily chosen value just in excess of the minimum. The curve of meter reading against distance shows a high degree of symmetry in the region of the minimum. When the number of minima in the tube allows, the positions of eight minima (four at the specimen end and four at the distant end of the tube) are measured. These data allow not only a convenient check on each other, but also make it possible to determine directly the wave-length of the sound, which enters into the calculation.

Thus for a sample of rock-wool (stillite, 5 lb./cu. ft.) 1″ thick at 1500 c/s., successive maxima and minima are shown in the following table. The maximum remains closely constant at 13·6 db. (the reference level is arbitrary, but the same throughout the measurements). The acoustic pressure is plotted directly against distance from the sample in figure 1, and extrapolation back to the sample shows that, with no dissipation in the tube, the minimum pressure would have a value of 0·99. The ratio $|p_{\max}|/|p_{\min}|$ of maximum to minimum pressure is, therefore, 4·83, i.e. 13·7 db.

The positions of eight minima are also shown in the table. By pairing of readings, four values for 2λ are obtained and lead to a value of λ of 23·00 cm. and to a corresponding velocity of sound of 345·0 m./sec. The position of the first minimum (d_1) can be obtained directly from the first reading, viz. 2·895 cm., from the sample. This value can be checked by means of the other figures

Position of minima of pressure and magnitude of maxima and minima for a typical sample of rock-wool

Stillite, 5·0 lb./ft.³, 1″ depth

Order N	Intensity $\lvert p_{max}\rvert/\lvert p_{min}\rvert$ (db.)	Distance from sample d_N(cm.)	2λ from preceding column (cm.)	$d_1=\left(d_N-(N-1)\frac{\lambda}{2}\right)$ (cm.)
1	0·0	2·895		2·895
	13·6			
2	0·2	14·39		2·89
	13·6			
3	0·5	25·89		2·89
	13·6			
4	0·75	37·39		2·89
	13·65			
5	1·0	48·89	45·995	2·89
	13·65			
6	1·25	60·395	46·005	2·895
	13·65			
7	1·45	71·895	46·005	2·895
	13·7			
8	1·65	83·395	46·005	2·895
			Mean 46·00	

since the successive readings d_N should differ from the first reading only by some known multiple $(N-1)$ of the measured half-wave-length. Thus the last column of figures in the table is found by subtracting from the measured values of d_N the appropriate multiple of the half-wave-length, and it is apparent that the agreement in the resulting values of d_1 is excellent.

Using the above data,

$$\lvert p_{max}\rvert/\lvert p_{min}\rvert = 4{\cdot}83 \text{ or } 13{\cdot}7 \text{ db.},$$

$$d_1 = 2{\cdot}895 \text{ cm., i.e. } d_1/\lambda = 0{\cdot}1259.$$

The correction to d_1 as calculated from (7) is 0·001 cm. The correction is not significant at this frequency. The corresponding values of $R/\rho c$ and of $X/\rho c$ are

$$R/\rho c = \quad 0{\cdot}399,$$

$$X/\rho c = -0{\cdot}927.$$

The length (86·5 cm.) of the tube used in the present apparatus is such that two or more minima can be located only for frequencies of 300 c/s. and above. Direct measurement of λ from the interval between minima is therefore impossible below 300 c/s., and λ must be found by other methods. Careful measurement of the wave-length, and hence of the velocity of sound, in the tube in the range of frequency 400 to 4000 c/s. has shown that the velocity C' follows closely the Helmholtz-Kirchhoff law,

$$C' = C\left(1-\frac{\sqrt{\nu'}}{2r\sqrt{\pi}}\cdot\frac{1}{\sqrt{f}}\right),$$

where C is the free-space velocity, r is the radius of the tube, f the frequency and ν' a function of the kinematic viscosity and diffusivity of the air (Rayleigh) equal numerically, for r in centimetres, to 0·577 at 23° C.

The usual procedure adopted with the apparatus is to measure C' accurately at one frequency (3000 c/s., at which many minima are available) and to apply a small correction calculated from the Helmholtz-Kirchhoff relation, at other frequencies. By this means the wave-length in the tube can be determined for frequencies below 300 c/s. and measurements of acoustic impedance can be made at 100 c/s., at which frequency the tube-length is only one-quarter of the wave-length.

§ 4. REDUCTION OF RESULTS

Apart from the corrections for attenuation, the values of $|p_{\max}|/|p_{\min}|$ and of d_1/λ are provided directly by the apparatus, and there remains the process of calculating the real and imaginary components of the impedance from them. This problem reduces to the solution of equations (1), (2) and (3). These equations can be solved algebraically, and thus it can be shown that

$$R/\rho c = \frac{2r}{1+r^2+(1-r^2)\cos 4\pi d_1/\lambda},$$

$$X/\rho c = \frac{-(1-r^2)\sin 4\pi d_1/\lambda}{1+r^2+(1-r^2)\cos 4\pi d_1/\lambda},$$

where

$$r = \tanh\gamma = \frac{|p_{\min}|}{|p_{\max}|}.$$

Numerical evaluation of $R/\rho c$ and $X/\rho c$ is , however, tedious, especially when results are to be worked out at several frequencies. It is convenient, therefore, to have graphical methods of solution available. In this respect the position is fortunate. H. J. Sabine (1942) has constructed charts with rectangular axes $R/\rho c$ and $X/\rho c$, on which are plotted families of curves of constant $|p_{\max}|/|p_{\min}|$ (in decibel units) and of constant d_1/λ. (He plots for D_1/D_2, where D_1 has the same significance as our d_1 and $D_2 = \frac{1}{2}\lambda$.) These charts are very useful for measurements at medium frequencies on materials of depth greater than 1 inch. For lower frequencies and small depths, e.g. where $R/\rho c$ may be of the order unity and $X/\rho c$ of the order -20, the range of curves is inadequate.

A second chart suitable for the calculation of $R/\rho c$ and $X/\rho c$ has been provided recently for the solution of a closely analogous problem in connection with radio wave-guides. Such a chart has been published by P. H. Smith (1939) and by Willis Jackson and L. G. H. Huxley (1944). In this chart, functions of $|p_{\max}|/|p_{\min}|$ and of d_1/λ are provided as polar co-ordinates, and curves of constant $R/\rho c$ and $X/\rho c$ are plotted thereon. The resulting families of curves corresponding to all values of $R/\rho c$ from zero to infinity and of all values of $X/\rho c$ from minus to plus infinity lie inside a circular boundary of finite radius. This type of chart is easily constructed from the data provided in the above references and is very convenient to use in practice.

Figure 6 shows the results of typical measurements of acoustical impedance obtained, in the manner described above, over the range 100 to 5000 c/s.,

of a 1″ thick specimen of rock-wool (stillite: 5 lb./ft.^{3}) backed by a substantially rigid piston.

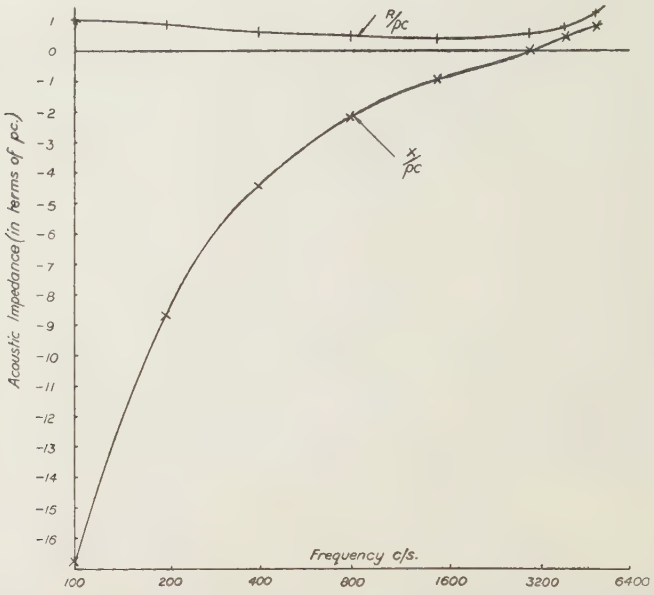


Figure 6. Typical measurements made with the apparatus of acoustic impedance of a sound-absorbing material.

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The author's thanks are due to Mr. C. P. Brittain for his services in the construction and testing of the apparatus, Dr. A. J. King for his interest in this work, Mr. B. G. Churcher, M.Sc., Manager of the Research Department, for authorization of the work, and Sir Arthur P. M. Fleming, C.B.E., Director of Metropolitan-Vickers Electrical Co., Ltd., for permission to publish this paper.

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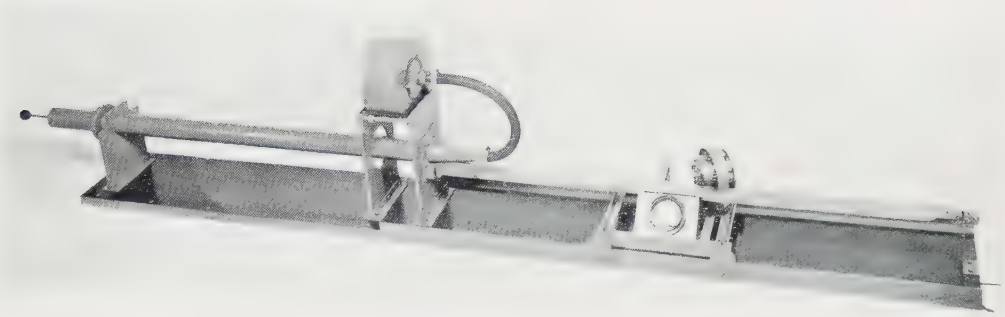
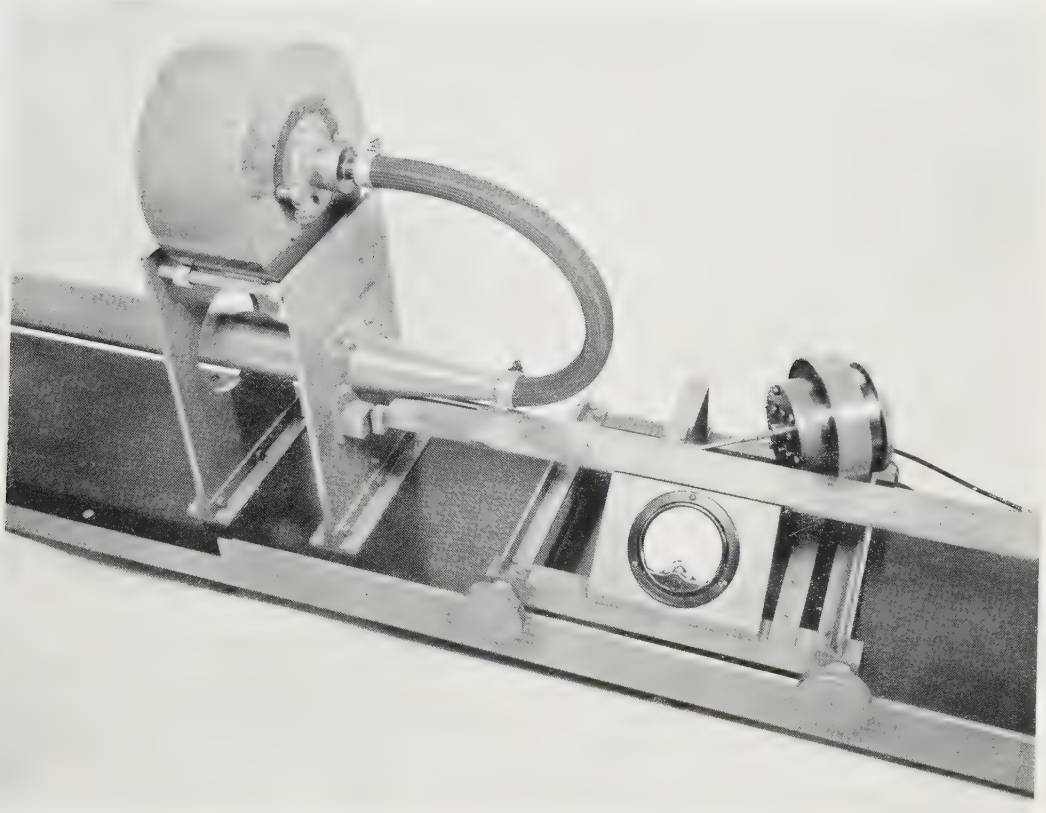


(*a*) General view of apparatus.



(*b*) View of driving end of apparatus and of microphone trolley.

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General view of reflection experiments.

AN EXPERIMENTAL INVESTIGATION OF THE REFLECTION AND ABSORPTION OF RADIATION OF 9-CM. WAVE-LENGTH.

By L. H. FORD and R. OLIVER,
National Physical Laboratory, Teddington

MS. received 21 *December* 1945

ABSTRACT. The paper, which was circulated as a confidential report to the Radio Research Board in October 1944, describes measurements on the reflecting power of various surfaces at a wave-length of 9 cm. and angles of incidence ranging from 80° to 45°; these included level and uneven bare ground, vegetation-covered ground, tap water and a 4% salt solution approximating to sea water. Specular reflection was found to occur only from very level surfaces; the absorptions of these surface media were measured, and from the combined measurements of reflection and absorption their electrical constants were derived.

Rough surfaces, either of bare ground or vegetation-covered ground, gave values of reflection coefficient in general agreement with the optical rule that regular reflection is only observed from an uneven surface if the product of the depth of the surface irregularities and the cosine of the angle of incidence is a small fraction of the wave-length. If this fraction exceeded $\frac{1}{2}\lambda$, the values of reflection coefficient measured were about 0·1.

The calculated values of reflection coefficient corresponding to dry soil, wet soil and sea water for angles of incidence varying from 0° to 85° are given in an appendix to the paper.

§ 1. INTRODUCTION

The reflection of radio waves has been investigated experimentally at a wave-length of 9 cm., using a wide range of reflecting surfaces; these included bare ground, both level and in ridges, ground covered with vegetation, fresh water, and salt water approximating in composition to sea water. When specular reflection occurs, the electrical constants of the material forming the reflecting surface can generally be estimated from measurements of the reflection coefficient. At the high frequencies involved in this investigation, however, the reflection coefficient is determined almost entirely by the dielectric constant; consequently reflection experiments, while providing a reasonably accurate indication of the dielectric constant, only enable a very rough estimate to be made of the conductivity of the surface material. The absorption in the material, on the other hand, is then dependent mainly on the conductivity. Experiments were therefore made on the absorption, at the same wave-length, of the materials of the various reflecting surfaces, from which the conductivities could be determined.

Since both reflection and absorption experiments are needed to obtain the electrical constants, the sequence adopted in the presentation of this paper is to describe each of the experimental methods before giving any of the results obtained from either, rather than to divide the paper into two distinct sections. The results of both reflection and absorption measurements are then given together, for each type of surface in turn.

§ 2. EXPERIMENTAL METHODS

A. *Measurement of reflection coefficients*

The measurement of the coefficient of reflection for radio-waves is simplified at centimetre wave-lengths by the possibility of using aerial systems which give beams of radiation which are so narrow that, except at angles within a few degrees of normal or grazing incidence, the signals due to the direct and the reflected rays can be measured separately by suitably adjusting the aerial. The magnitude of the reflection coefficient is thus obtained from the ratio of two measurements of field strength; no indication of its phase angle is obtained.

A diagrammatic representation of the arrangement of the apparatus is shown in figure 1, while the plate shows a photograph of an experiment. The two towers were constructed of tubular scaffolding, and were 15 metres (50′) apart, with platform levels of 1·7 m. (5·5′), 4·3 m. (14′) and 6·7 m. (22′) above ground. The

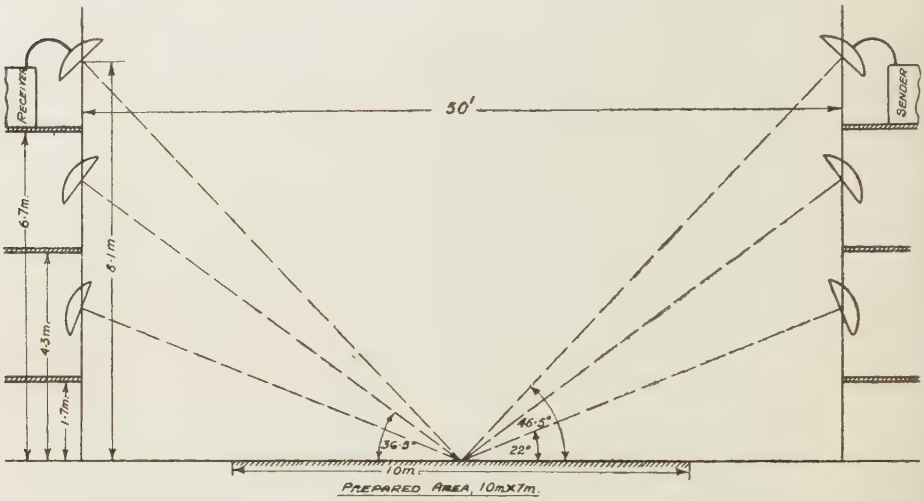


Figure 1. Diagrammatic representation of ground reflection experiments.

1·2-metre diameter aperture parabolic mirrors, each of which was used with a half-wave dipole aerial at the focus, were so mounted that the aerials were 1·4 metre above the platform level, thus giving angles between the reflected ray and ground of 22°, 36°·5 and 46°·5 (angles of incidence of 68°, 53°·5 and 43°·5). Between the two towers a symmetrically disposed area of ground with sides 7×10 metres was prepared to obtain the several types of reflecting surface used. For experiments on water, a National Fire Service canvas dam, giving a water surface of about 5 metres diameter, was placed symmetrically between the towers.

The sender consisted of a CV. 67 valve oscillator mounted directly on the back of one of the paraboloids, and a crystal monitor was used to ensure constancy of output during each set of observations. Two receivers were used: one (Saxton and Grace, 1946) was a bolometer mounted directly behind the receiving paraboloid, the other was a field-strength measuring set (Grace, 1946). When using the bolometer, reflection coefficients greater than about 0·2 could be obtained with an instrumental accuracy of about $\pm 0{\cdot}01$, from the square root of the ratio of the

values of the power received with a direct and with a reflected signal. When using the field-strength measuring set, the ratio of the direct and reflected received signals, in decibels, could be measured with an accuracy of about $\pm 0{\cdot}5$ db. if less than 6 db., and an accuracy of about ± 1 db. if greater than 6 db. Thus the bolometer was considerably more sensitive than the field-strength measuring set, except under conditions where the reflected signal was small.

The procedure followed at each level was to adjust the sending and receiving mirrors in turn until a maximum signal could be measured from the direct ray; next to readjust the mirrors for a maximum signal from the reflected ray, and finally to repeat the measurement of the direct signal to verify that the sender output had remained constant.

Apart from instrumental inaccuracies in measurement, there are a number of possible sources of error in the results obtained. Experiment showed that the polar diagrams of the mirrors were such that any signal due to the direct ray, when measuring the reflected ray, would be more than 40 db. down, which virtually eliminated one potential source of error. Experiment also showed that a distance of 15 metres between the mirrors was somewhat less than the minimum distance at which the transmission law holds, that the received signal varies inversely as the distance from the sender. A careful series of measurements was therefore made to determine the variation of received signal with distance from the sender over the range concerned, and the values thus obtained were used in place of the geometrical values when correcting the measurements of reflected signal for the path difference between the direct and reflected rays. A slight, but perceptible, interaction could, moreover, be detected between the receiver and sender mirrors, due to reflection from receiver to sender mirror when the two were directed at each other; calculation showed that this was unlikely to cause an error of more than 0·01 in the reflection coefficient. The agreement obtained between measurements at three different angles of incidence, and consequently three path differences, under conditions of specular reflection, makes it improbable that any of these sources of error was serious; consequently, the reflection coefficients measured with bolometer are believed to be accurate to $\pm 0{\cdot}02$. Had the distance between the towers been made great enough to reduce all the sources of error mentioned to negligible proportions, both the height of the towers and the area of ground requiring preparation would have become undesirably large.

To check the reliability of the method, a level platform of planks was set on the ground midway between the towers, and on it were placed six 1×2-metre iron sheets, making an area of 3×4 metres. The reflection coefficient of this was measured at angles of incidence of $53°{\cdot}5$ and $43°{\cdot}5$. The reflection coefficients obtained varied between 0·95 and 1·03, with a mean value of 0·98. The observations were made with the field-strength set, and the possible instrumental error was consequently $\pm 0{\cdot}5$ db. The values obtained, therefore, are in sufficiently good agreement with the expected value of unity to indicate that none of the possible sources of error in the experimental method was serious.

B. *Measurements of absorption*

The absorption of radiation by a screen of the reflecting medium used in the previous experiments was measured directly by interposing a filled container

between sender and receiver. The arrangement used is shown diagrammatically in figure 2. The sender was exactly as used in the reflection experiments, and was set up about 6 metres away from a table on which stood the test sample and the receiver. The latter consisted of a horn in which was a bolometer, forming the field-strength measuring device described by Saxton and Grace. A container consisting of a wooden trough was made having ends of 6 mm. plate glass, 46 cm. square, one of which was fixed and the other movable, thus forming a container in which a sample 46 cm. square and of any desired thickness up to 30 cm. could be tested. This container could be interposed between the sender and the receiving horn.

The experimental procedure was to measure the field strength at the horn when the transmission path from the sender was unobstructed, and then to repeat the measurement with the container filled with a chosen thickness of the substance under test, placed symmetrically to obscure the horn from the sender. Successive measurements were made with the container at various distances from 1 to 6 cm.

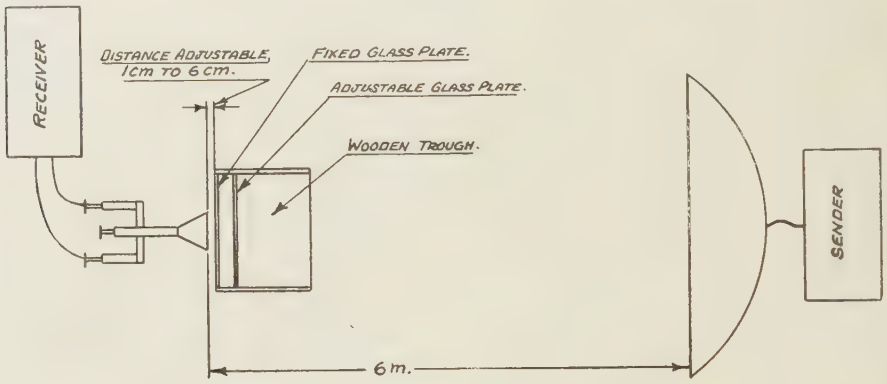


Figure 2. Diagrammatic representation of absorption experiments

from the aperture of the horn. In most cases considerable variations were found to be present in the received signal as the distance was varied, due, presumably, to reflection from the end of the container nearest the horn, and the mean value over a range of distance equal to half a wave-length was taken as the true transmitted signal. The thickness of the obstruction was then altered by adjusting the movable glass end-plate and refilling the container, and the experiment repeated. Thus the attenuation in a known thickness of the material under test could be obtained without reference to the properties of the container.

The method of measurement was subject to limitations; if the thickness of material under test were too small, multiple reflections between the two end plates would be a serious source of error, while if the thickness were too great the accuracy of measurement would be inadequate. With thicknesses of material such that the attenuation was between about 5 and 15 db., however, the measurements are believed to be accurate to $\pm 0{\cdot}5$ db.

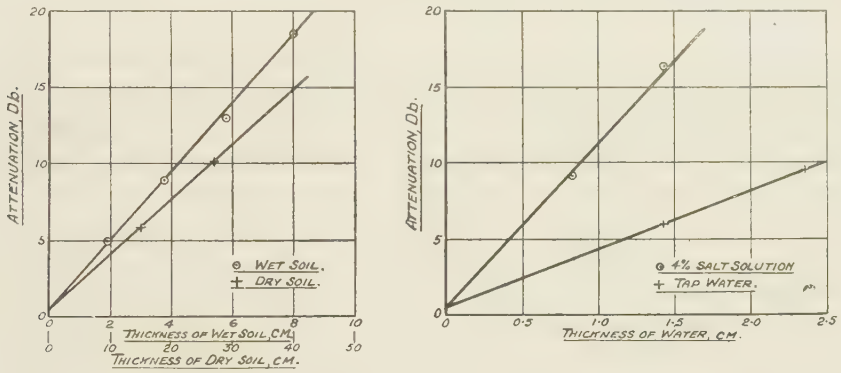
§ 3. EXPERIMENTAL RESULTS: SPECULAR REFLECTION

A. *Reflection and absorption from bare ground*

An area 7×10 metres midway between the towers was cleared of turf, forked over to a depth of about 10 cm., carefully raked level, and finally smoothed by

dragging a plank across it. Preliminary experiments showed that the surface thus obtained gave specular reflection. This conclusion was based on the fact that a single value of dielectric constant, K, and conductivity, σ, could be chosen which gave calculated values of reflection coefficients which, at all angles and with both horizontal and vertical polarizations, agreed within the accuracy of experiment with the measured values. Curves connecting the reflection coefficient with K and σ, of the type described by McPetrie (1938), were drawn for each of the angles of incidence available to simplify the choice of the appropriate values of K and σ to fit any given set of test results. It quickly became apparent that the dielectric constant rather than the conductivity determined the reflection coefficient, and that changes in the wetness of the ground caused large variations in the value obtained. The decision was therefore made to concentrate on the two extreme conditions of humidity.

The penetration of the wave into the ground on reflection with centimetre wave-length radiation is only a few centimetres, which is much less than occurs at



(*a*) Soil. (*b*) Water.

Figure 3. Attenuation of $\lambda = 9$ cm. radiation in various media.

the longer wave-lengths; consequently extreme conditions, particularly of dry ground, will be more often encountered with centimetre waves than with the longer wave-lengths.

An immediate consequence of the relative unimportance of the conductivity in determining the reflection coefficient is that the change in phase on reflection is about 180° for horizontally polarized waves at all angles of incidence; it is about 180° for vertically polarized waves at angles of incidence appreciably greater than the Brewster angle, and at angles of incidence appreciably less than that angle there is no appreciable phase change on reflection.

(a) *Dry ground.* A prolonged period during May 1944 in which no rain fell gave an opportunity to measure the ground reflection under conditions where the surface layer was as dry as is ever probable; no trace of moisture was apparent, and the top surface was almost as loose in consistency as sand.

The decision to measure the ground absorption was not taken until some time after the reflection tests had been made. Consequently, to obtain ground in as nearly as possible the same condition, a sample of surface soil was placed under cover and left for about the same period as had elapsed without rain before the reflection test. The results of the absorption tests on dry soil are shown in figure 3 (*a*), and indicate an attenuation of 36 db. per metre.

The most suitable values of K and σ to fit both reflection and absorption measurements were

$$K=2, \qquad \sigma=3\times10^{8} \text{ e.s.u.}$$

The reflection coefficients calculated from these values, together with the measured values, are given in table 1.

Table 1. Reflection coefficients for very dry ground, $\lambda=9$ cm.

Angle of incidence (degrees)	Vertical polarization		Horizontal polarization	
	Calculated	Observed	Calculated	Observed
68	0·18	0·20	0·48	0·47
53·5	0·015	0·03	0·33	0·33
43·5	0·08	0·09	0·26	0·27

(b) *Wet ground.* The preliminary experiments showed that the value of K corresponding to the reflection coefficients usually encountered varied between 5 and 15, according to the degree of wetness of the ground. The opportunity was taken during a rainy spell in February to obtain a series of measurements immediately after a twelve-hour period of steady rain, when the ground was well saturated. The field-strength measuring set was used, so that the results obtained were of less accuracy than the others given in this report for conditions of specular reflection, all of which were taken using the bolometer. Accordingly, a further series of measurements were made in which, since the weather conditions were not suitable, the ground was kept wet artificially. For this purpose a spray branch of the type used to deal with electrical fires was used with a fire hose to water the site, and the equivalent of about four inches of rain was delivered at intervals over a period of 48 hours. A slight further watering was given about half an hour before each test. The resulting condition of the ground was such that it was saturated with water, but was not waterlogged, i.e. no puddles were present on the surface.

To obtain a similar sample of soil for absorption measurements the container was filled with soil and water, and allowed to stand until the surplus water had drained off. It was found that this method gave closely repeatable results, and the assumption that the sample in the container was then under the same conditions of humidity as the ground during the reflection experiments seemed justified. The results of the absorption tests on saturated soil are shown in figure 3 (*a*), and indicate an attenuation of 220 db. per metre.

The most suitable values of K and σ to fit both reflection and absorption measurements were

$$K=24, \qquad \sigma=6\times10^{9} \text{ e.s.u.}$$

The reflection coefficients calculated from these values and the measured values are given in table 2; calculated values assuming $K=25$, $\sigma=0$, are also included to demonstrate the comparatively small contribution made by the conductivity to the reflection coefficient.

The ground surface on which the above experiments were made is composed of a fine sandy loam. Other types of ground might give appreciably different results.

The conductivities of both dry and wet soil were measured at a frequency of 1 Mc/s. ($\lambda = 300$ m.) by the Electricity Division, N.P.L., using a laboratory method. The values obtained were 3×10^5 and 8×10^7 e.s.u. for dry and wet soil respectively. These agree with the conductivities given by Smith-Rose (1933) for the soil at Teddington in extreme conditions of low and high humidity, and

Table 2. Reflection coefficients for saturated ground, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization				Horizontal polarization			
	Calculated		Observed		Calculated		Observed	
	$K=24$ $\sigma=6\times10^9$	$K=25$ $\sigma=0$	Heavy rain	Watered by hose	$K=24$ $\sigma=6\times10^9$	$K=25$ $\sigma=0$	Heavy rain	Watered by hose
68	0·31	0·31	0·28	0·32	0·85	0·86	0·90	0·86
53·5	0·50	0·50	0·50	0·50	0·78	0·79	—	0·78
43·5	0·57	0·57	0·58	0·58	0·74	0·75	0·72	0·74

thus show that the site used was not abnormal. Smith-Rose and McPetrie (1934) have measured the constants of the soil at Teddington by a laboratory method at a frequency of 200 Mc/s. ($\lambda = 1{\cdot}5$ m.), and they obtained values of $K = 22$ and $\sigma = 3 \times 10^9$ e.s.u., and of $K = 4$ and negligible conductivity, under extreme wet and dry conditions respectively.

B. *Reflection and absorption from water*

Tests on the reflection of radiation of 9 cm. wave-length were made on fresh and salt water by placing a circular canvas dam, lent for the purpose by the National Fire Service, midway between the towers. This was filled to a depth of about 30 cm. with tap water or with a 4% solution of coarse salt, approximating to sea water. The diameter of the base of the dam was 5·5 metres (18 feet), and that of the water surface about 5 metres. The area available for reflection was consequently rather small, although experience had shown that it would probably be sufficient to give reasonably reliable results. The accuracy was, however, less than that of the ground-reflection experiments described above.

The absorption experiments were made using the same container as before, the cracks between the end plates and sides being caulked with Plasticine to render them watertight. The thickness of liquid under test was obtained by measuring the volume necessary to fill the container in each case.

(a) *Tap water.* The results obtained from the absorption tests on tap water are shown plotted in figure 3 (*b*), and indicate an attenuation of 380 db. per metre.

The most suitable values of K and σ to fit both reflection and absorption experiments were

$$K = 80, \qquad \sigma = 2 \times 10^{10} \text{ e.s.u.}$$

The reflection coefficients calculated from these values, together with the measured values are given in table 3.

(b) 4% *salt solution.* The results from the absorption tests on a solution of 4% coarse salt in tap water are shown plotted in figure 3 (*b*), and indicate an attenuation of 1100 db. per metre.

Table 3. Reflection coefficients for tap water, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization		Horizontal polarization	
	Calculated	Observed	Calculated	Observed
70	0·51	0·51	0·92	0·90
55	0·69	0·67	0·88	0·88
44·5	0·73	0·70	0·85	0·83

The most suitable values of K and σ to fit both reflection and absorption experiments were

$$K = 80, \qquad \sigma = 5{\cdot}5 \times 10^{10} \text{ e.s.u.}$$

The reflection coefficients calculated from these values, together with the measured values, are given in table 4.

Table 4. Reflection coefficient for 4% salt solution, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization		Horizontal polarization	
	Calculated	Observed	Calculated	Observed
70	0·54	0·56	0·93	0·92
55	0·71	0·71	0·89	0·89
44·5	0·74	0·73	0·86	0·84

When making the absorption measurements on all types of material, observations were made using both radiation polarized parallel to and perpendicular to the edge of the container. It was found that a consistent difference of about 0·5 db. was present between the results with the two polarizations. A test on the empty container with the end plates removed showed that the container slightly increased the gain of the horn, and that this increase differed between the polarizations by the amount necessary to account for the discrepancy with the filled container. The experimental points plotted on the curves of figure 3 are all mean values of the results obtained with each polarization.

C. *Reflection from turf-covered ground*

A few measurements were made on the reflection coefficient of ground covered with short grass. The results obtained showed that, in general, specular reflection was not present. An exception was found when observations were made on a recently cut and rolled cricket wicket. The distance between the mirrors during the test was 21·5 metres and the height of the aerials 1·85 metres each, giving an angle of incidence for the reflected radiation of 80°. The results obtained are given in table 5, which also gives calculated values of reflection coefficient based on suitable assumed values of K and σ. Values obtained on an adjacent cricket wicket on which the grass had been allowed to grow to a height of about 2 cm. are also included in the table.

The value of K corresponding to the wet condition is lower than might be expected from the bare-ground experiments, since the site appeared to be well

soaked by the previous night's rain. No opportunity occurred to take measurements under conditions such that the ground was certainly saturated. Accordingly,

Table 5. Reflection coefficients of level grass-covered ground (Cricket wicket), $\lambda = 9$ cm. Angle of incidence, 80°.

Conditions	Vertical polarization		Horizontal polarization		Assumed values	
	Calculated	Observed	Calculated	Observed	K	σ (e.s.u.)
Cut very short and rolled, dry. (No rain for at least 7 days)	0·47	0·46	0·79	0·79	3	5×10^9
Cut very short and rolled, wet. (Several hours' rain in previous night)	0·36	0·37	0·86	0·86	6	5×10^9
Grass about 2 cm. high, wet. (Same day as previous test)	0·36	0·51	0·86	0·83	6	5×10^9

although the results appear to indicate that the value of K for ground covered with turf is less than that of bare ground of the same humidity, this conclusion cannot be given with certainty.

D. *Summary of specular reflection results*

The values of K and σ appropriate to the various reflecting surfaces used in the investigation are brought together, for convenience, in table 6, which also gives the attenuation in the medium.

Table 6

Nature of medium	K	σ (e.s.u.)	Attenuation (db./metre)
Very dry sandy loam	2	3×10^8	36
Saturated sandy loam	24	6×10^9	220
Tap water	80	2×10^{10}	380
4% solution of coarse salt	80	$5{\cdot}5\times10^{10}$	1100
Dry turf	3	(5×10^8) (estimated)	(50) (calculated)
Wet turf	6	(10^9) (estimated)	(80) (calculated)

The values of attenuation and conductivity found for salt and tap water are somewhat lower than those previously obtained, using a different method, by Turner for a wave-length of 10 cm., but agree with his results in showing that the ratio between the conductivities of salt and tap waters is much less than at low frequencies. The value of 80 for the dielectric constant of water is not precise; a figure of 75 would fit the experimental results equally well.

§ 4. EXPERIMENTAL RESULTS: DIFFUSE REFLECTION

At centimetre wave-lengths, specular reflection is likely to be the exception rather than the rule, since the reflecting surfaces normally encountered are neither level nor free of vegetation. Experiments have been made to determine the reflection coefficient of level ground covered with vegetation, and of uneven ground.

A. *Effect of vegetation*

The results given in the last line of table 5 show the effect of grass about 2 cm. high on the reflection coefficient of level ground at an angle of incidence of 80°. The effect of the grass was to decrease the reflection coefficient for horizontally polarized waves, and to increase it for vertically polarized waves. The latter result was most unexpected, but the phenomenon was observed on several occasions (see also table 7). On longer grass the reflection coefficient for both polarizations was reduced; unmown grass in May, approximately 30 cm. high, gave coefficients of 0·19 with horizontally and 0·13 with vertically polarized radiation, at an angle of incidence of 80°.

Table 7. Reflection coefficients of vegetation-covered wet ground, $\lambda = 9$ cm.

Appearance of site, and height of vegetation		Angle of incidence of radiation, θ					
Appearance	Height (cm.)	$\theta = 68°$		$\theta = 53°{\cdot}5$		$\theta = 43°{\cdot}5$	
		V	*H*	*V*	*H*	*V*	*H*
Bare	0	0·32	0·86	0·50	0·78	0·58	0·74
True leaves beginning to form, ground visible	3–4	0·40	0·50	0·44	0·55	0·47	0·56
Dense clumps, ground showing in places	9–12	0·18	0·65	0·23	0·58	0·33	0·49
Ground almost obscured	20–25	0·06	0·32	0·10	0·39	0·17	0·41
Ground completely obscured	35–45	0·04	0·19	0·05	0·26	0·11	0·28

The main experimental results were taken on the site between the towers, on which the bulk of the previous results on ground reflection had been made. The levelled area was sown fairly thickly with mustard seed (very approximately 1000 germinations per square metre) to obtain a quick-growing carpet of vegetation. To ensure constancy of ground conditions, the site was watered thoroughly before each test as described in § 3, A (b). The results are given in table 7 for both vertical, *V*, and horizontal, *H*, polarizations.

The site at the conclusion of the test was covered completely with a dense carpet of vegetation, the top surface of which was comparatively level. It is probable that the reflection coefficient would be even lower with a less uniform ground covering. These results are in general agreement with those given by other investigators.

B. *Effect of uneven ground*

The effect of an uneven ground surface was investigated by raking the bare surface of the site into a series of uniform ridges. When planning the experiments,

the possibility of obtaining information applicable to the surface of the sea was borne in mind, and the system of ridges was made to simulate waves. Tests were made with these running along, across, and at 45° to the direction of transmission, using distances of 0·6 and 1·2 metres between successive crests, and various depths of between 5 and 16 cm. between crest and trough. Table 8 shows the reflection coefficients observed when the ridges were at 45° to the direction of transmission, while table 9 gives those obtained when the ridges were either along or across the direction of transmission. In these tables D gives the distance (cm.) between the crests of successive ridges, and h the mean depth (cm.) between crest and trough, while the column marked "Level" gives the estimated reflection coefficient for level ground of the same humidity as that under test.

Table 8. Reflection coefficients of ground ridged at 45° to the direction of transmission, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization				Horizontal polarization			
	Level	$D=60$ $h=14$	$D=120$ $h=10$	$D=120$ $h=5$	Level	$D=60$ $h=14$	$D=120$ $h=10$	$D=120$ $h=5$
68	0·08	0·07	0·09	0·13	0·65	0·14	0·18	0·30
53·5	0·13	0·04	0·05	0·07	0·51	0·04	0·06	0·16
43·5	0·22	0·04	0·04	0·04	0·45	0·07	0·06	0·10

Table 9. Reflection coefficients of ground ridged along or across the direction of transmission, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization				Horizontal polarization			
	Level	Along $D=60$ $h=14$	Across $D=60$ $h=16$	Across $D=120$ $h=12$	Level	Along $D=60$ $h=14$	Across $D=60$ $h=16$	Across $D=120$ $h=12$
78	0·23	0·2	0·1	0·3	0·86	0·4	0·2	0·4
68	0·06	0·03	0·05	0·12	0·76	0·07	0·10	0·18
53·5	0·28	0·03	0·02	0·06	0·64	0·10	0·12	0·16
43·5	0·36	0·04	0·08	0·08	0·58	0·04	0·08	0·14

The results show clearly how small an irregularity in the ground surface is sufficient to prevent specular reflection; the reflection coefficient, moreover, becomes erratic with respect to angle of incidence when it has fallen below a value of about 0·1. The values given for level ground, although consistent among themselves, only relate very approximately to the state of the ground in the ridged condition, since the length of time necessary to make all the measurements, several days, was sufficient to allow marked changes in ground humidity. The difference between the values for level ground given in table 8 and those in table 9 are due to the observations recorded in the former table having been taken later in the year than those recorded in table 9, when the ground was appreciably drier. Values are given in table 9 for an angle of incidence of 78°. These were obtained with the mirrors at ground level about 14 metres apart. The accuracy

in these circumstances was not considered sufficient for measurements under conditions of specular reflection, but is adequate to give an indication of the reflection present from uneven ground.

The presence of a pattern in the signal reflected from ridged ground had been expected at the start of this investigation. Measurements were therefore made in which the mirrors, although always maintained at a distance of 15 metres apart, were moved so that the point of reflection to be expected from geometry was successively at different parts of the ridge system. Thus, when working on ridges running across the direction of transmission, the mirrors were moved together in the line of transmission to a number of successive positions so that the geometrical point of reflection varied from being on the crest of a ridge to being in the trough between two ridges. The somewhat unexpected result was obtained that the reflection coefficient remained substantially constant, at each angle of incidence, provided that the mirrors were set each time to obtain a signal maximum. The variable strength of reflected signal obtained from the constantly changing surface of the sea, therefore, may be partly due to the impossibility of maintaining a mirror, at every instant, in optimum adjustment. With ridges 120 cm. apart, running along the direction of transmission, it was found, however, that the reflected signal with the geometrical point of reflection on the crest of a ridge was appreciably greater than when the point of reflection lay in a trough.

When working on ridged ground, multiple reflections were, in general, observed. Thus, when the ridges were across the direction of transmission, in addition to a maximum of reflected signal obtained with the mirrors set at approximately the elevations indicated by geometry, further maxima could be obtained by adjusting the mirrors so that one had a greater and one a less angle of elevation than the geometrical. Three maxima of received signal of approximately equal value, but having different settings of mirror elevations, were not unusual. With the ridges running along the direction of transmission, a number of maxima of received signal could similarly be obtained by suitable adjustments of the azimuth settings of the mirrors.

The experiments on ridged ground described above were made on a surface of comparatively low reflection coefficient, and some similar experiments were therefore made on a surface of greater reflecting power. For this purpose a mat of 1·2 cm. ($\frac{1}{2}''$) mesh galvanized wire netting was laid between the towers, and its surface formed into a series of ridges. The material was somewhat intractable, but by pegging it to the ground at frequent intervals in the troughs, a system of ridges closely similar to those used for the ground experiments was obtained. The results obtained are given in table 10.

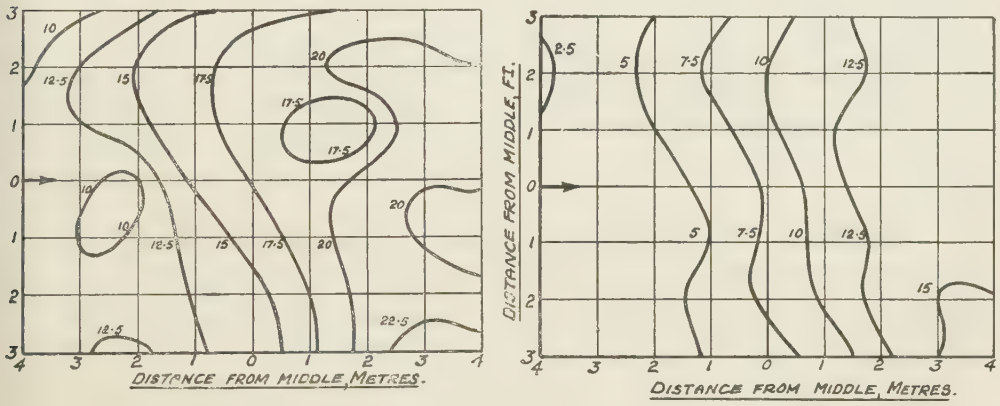
The reflection coefficients were notably greater than those for similarly ridged ground, but bore much the same ratio to the level values as held in the case of bare ground. Absorption tests showed that the wire netting transmitted 0·57 or 0·53 of the radiation incident normally on it, according as the electric vector was polarized parallel or perpendicular to the edge of the netting. This high proportion transmitted accounts for the comparatively low reflection coefficient with vertical polarization when the netting was level, and raises some doubts whether the result as a whole can be taken as typical for a surface of high reflecting power.

Table 10. Reflection coefficient of wire netting ridged at 45° to the direction of transmission, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization			Horizontal polarization		
	Level	$D=60$ cm. $h=10$ cm.	$D=120$ cm. $h=15$ cm.	Level	$D=60$ cm. $h=10$ cm.	$D=120$ cm. $h=15$ cm.
68	0·80	0·20	0·25	0·99	0·25	0·31
53·5		0·20			0·31	
43·5		0·23			0·26	

C. *Other observations on rough ground*

When the investigation started, in February 1944, the site between the towers consisted of rough unmown land, having variations in ground level of about ±7 cm. on the mean over the area under test, and covered with a dense matted layer of grasses about 30 cm. high, with dead stalks rising to a height of more than 60 cm. The reflection coefficient of this surface was measured, and the grass then mown as short as the roughness of the ground permitted. The ground contour of the central 6 × 8 metres of the area between the towers was measured, and a



(*a*) Before removing turf. (*b*) After removing turf.

Figure 4. Ground contour maps of site.
Contours every 2·5 cm. above datum. Direction of transmission →

contour map is given in figure 4 (*a*). The reflection coefficient of this surface was also measured. The turf was next removed, and in doing this some of the ground irregularities were removed. A contour map of the site in this condition is given in figure 4 (*b*). Reflection coefficients were again measured on this surface.

The results of these tests are given in table 11, in which are also given the estimated reflection coefficients for level ground of the same humidity.

§ 5. DISCUSSION OF RESULTS AND CONCLUSIONS

The investigation has provided a fairly comprehensive outline of the phenomena associated with the reflection of radiation of 9 cm. wave-length, with two limitations. The first of these is the possibility of a scale effect when the results of

the experiments on diffuse reflection, made on an area of the order of 20 square metres, are applied to cases of transmission over considerable distances, when the area contributing to the reflected signal will be much larger. The second limitation is that no results have been obtained with angles of incidence greater than 80°, since the method used is clearly not applicable to cases where the angular separation of the direct and reflected rays is small.

The general result of the experiments on diffuse reflection is to confirm the optical rule that, for regular reflection for a rough surface, the depth of the surface irregularities multiplied by the cosine of the angle of incidence must be a small

Table 11. Reflection coefficients of rough ground, $\lambda = 9$ cm.

Angle of incidence (degrees)	Vertical polarization				Horizontal polarization			
	Level (estimated)	Long grass	Short grass	Bare	Level (estimated)	Long grass	Short grass	Bare
68	0·20	0·08	0·20	0·23	0·82	0·12	0·74	0·81
53·5	0·40	0·06	0·17	0·27	0·73	0·12	0·43	0·50
43·5	0·48	0·06	0·16	0·26	0·68	0·03	0·36	0·46

fraction of the wave-length. The results of tables 8 and 9 indicate that when this fraction is about 1/5, the reflected ray may be reduced to about one half its proper value, and that when the fraction is 1/2 a reflection coefficient greater than 0·1 is improbable. There is no reason to suppose that these relationships will be greatly altered by either an increase in the area of reflecting surface or in the angle of incidence.

The following conclusions on the reflection and absorption of radiation of 9 cm. wave-length can be drawn from the experiments described above:—

(1) Except at grazing incidence, specular reflection will be unusual, since it can only occur at very level surfaces which are free of vegetation.

(2) The reflection coefficient of bare ground when specular reflection occurs will usually lie between the values computed by assuming negligible conductivity and a dielectric constant of either 5 or of 15, according to the wetness of the surface layer.

(3) The change in phase on reflection is about 180° for vertical polarization at angles of incidence appreciably greater than Brewster's angle, and negligible at angles of incidence appreciably less than Brewster's angle.

(4) The change in phase on reflection is about 180° for horizontal polarization at all angles of incidence.

(5) For a very dry sand loam the attenuation is 36 db. per metre, the conductivity 3×10^8 e.s.u., and the dielectric constant 2.

(6) For the same ground when saturated with water the attenuation is 220 db. per metre, the conductivity 6×10^9 and the dielectric constant 24.

(7) For tap water the attenuation is 380 db. per metre, the conductivity 2×10^{10} and the dielectric constant between 75 and 80.

(8) For a 4% salt solution, approximating to sea water, the attenuation is 1100 db. per metre, the conductivity $5{\cdot}5 \times 10^{10}$ and the dielectric constant between 75 and 80.

(9) Specular reflection ceases to occur when the surface becomes uneven (as would be predicted by optics); and when the product of the depth of surface irregularity and the cosine of the angle of incidence is $\lambda/2$ or more, the reflection coefficient is unlikely to exceed 0·1 for all polarizations and angles of incidence.

(10) A layer of vegetation on level ground prevents specular reflection, even if it is only a few centimetres high; in this case the reflected signal with vertical polarization may be increased at high angles of incidence.

(11) An appreciable thickness of vegetation reduces the reflection coefficient at all angles of incidence to a value considerable below that obtained by specular reflection from the ground surface; if the layer of vegetation is more than 30 cm. high, the reflection coefficient is unlikely to exceed 0·2 at angles of incidence less than 80°.

(12) The probable value of reflection coefficient for rough ground covered with vegetation is about 0·1, except in the neighbourhood of grazing incidence.

§ 6. ACKNOWLEDGMENTS

The authors wish to express their thanks to the National Fire Service for the loan of equipment, and to acknowledge the help and advice which they received from Dr. R. L. Smith-Rose in the planning of the investigation. The work was carried out as part of the programme of the Radio Research Board, and this paper is published by permission of the Department of Scientific and Industrial Research.

APPENDIX

Calculated reflection coefficients are given in table 12 for surfaces consisting of very dry ground, very wet soil, and sea water. The values for fresh water will only differ by about 0·03 from those for sea water.

Table 12. Calculated reflection coefficients, $\lambda = 9$ cm.

Angle of incidence (degrees)	Very dry soil		Very wet soil		Sea water	
	Vertical polarization	Horizontal polarization	Vertical polarization	Horizontal polarization	Vertical polarization	Horizontal polarization
0	+0·17	−0·17	+0·67	−0·67	+0·81	−0·81
20	+0·16	−0·19	+0·65	−0·68	+0·80	−0·82
40	+0·10	−0·24	+0·60	−0·73	+0·76	−0·85
60	−0·06	−0·38	+0·42	−0·82	+0·66	−0·90
70	−0·21	−0·51	+0·27	−0·87	+0·54	−0·93
80	−0·49	−0·71	−0·08	−0·93	+0·26	−0·97
85	−0·71	−0·84	−0·39	−0·96	−0·11	−0·98

In the table a + sign indicates that the phase change on reflection is very small, and a − sign indicates a phase change of about 180°.

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NOTE ON THE BEHAVIOUR OF ZINC SULPHIDE PHOSPHORS UNDER CONDITIONS OF PERIODIC EXCITATION

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ABSTRACT. The behaviour of luminescent solids under conditions of periodic excitation has been discussed theoretically and predictions made on the basis of simple models for the electronic processes accompanying the emission of luminescent radiation. The significant features of the luminescence intensity as a function of time are the shift in phase with respect to the exciting radiation and the ratio of maximum to minimum emitted intensities. The variation of these parameters with intensity and period of excitation can be used to distinguish between various mechanisms for the luminescence process. Experiments are described, using a cathode-ray-oscillograph technique, illustrating the application of the method to zinc sulphide and zinc cadmium sulphide phosphors. The general conclusions are that these phosphors do not possess any of the characteristics associated with the monomolecular mechanism, but show a semi-quantitative agreement with the requirements of a simple ionization-recombination mechanism. Deviations from theory are ascribed to the complexity of these phosphors, which are activated by more than one type of activator atom.

§1. INTRODUCTION

MOST present-day applications of luminescent substances involve excitation by periodic stimuli, under which conditions phosphorescence is usually a desirable property; for example, the flicker in A.C.-operated fluorescent discharge lamps is reduced by the use of luminescent coatings exhibiting phosphorescence. The behaviour of phosphors when subjected to periodic excitation has been studied very little, and no theoretical treatment of the effect has been attempted. It has been observed, however, that the wave-form of the total light output from a fluorescent discharge lamp shows a slight advance in phase over the excitation wave-form (Davies, Ruff and Scott, 1942; Uyterhoeven and Zecher, 1938). It was considered, therefore, that an experi-

mental study of the wave-form of the luminescent radiation alone might give some indication of the laws governing the effect and eventually provide some data concerning the mechanism of the luminescent process. This paper describes a limited experimental study of the effect, and compares the results with predictions based on a theoretical treatment. The results can be considered only as preliminary, but they do serve to indicate the possibilities of this method, particularly with regard to the mechanism of the luminescent process.

Wood (1921), Gottling (1923) and Szymanowski (1935) studied certain luminescent materials by a method similar in principle to that described here, but based on the use of the electro-optical Kerr effect for the measurement of very short time intervals. Both Wood and Gottling interpreted their results as indicating a dark interval between absorption of radiation and emission of luminescence, and it remained for Gaviola (1925) to show that this interval corresponded to the shift in phase of the luminescence intensity-time function with respect to that of the high-frequency spark excitation.

§ 2. THE BASIC LUMINESCENCE PROCESSES AND PERIODIC EXCITATION

It is to be expected that the behaviour of a phosphor under periodic excitation will depend markedly on the nature of the mechanism of the decay of luminescence. This mechanism is, in general, complicated (Randall and Wilkins (1945 a and b), Garlick and Wilkins (1945), Lord, Rees and Wise (1946)), but can be broadly classified into two main types: that in which electrons do not leave the luminescent centres, and that in which they do. In this preliminary investigation we have therefore examined the differential equations describing the luminescent emission of radiation under periodic excitation in the simplest case of each of the two types, and have noted certain significant differences in the solutions. The simplest case of the first type is that which for constant excitation leads to a simple exponential decay law. The simplest case of the second type is that which for constant excitation leads to a simple bimolecular decay law. More complicated cases of the two types are then discussed in a semi-quantitative way in § 3.

All periodic functions $I(t)$ representing the time-dependence of the intensity of the exciting radiation can, if necessary, be expressed as Fourier series, but it is evident that the simplest possible function, namely, that describing a sinusoidal variation, should be employed in the initial theoretical considerations, and also, if it is proved to be practicable, in any experimental work which attempts to establish the general predictions of the theory.

In the simplest case, in which the electronic transitions responsible for the absorption and emission of radiation in phosphors are all internal, i.e. in the case leading, for constant excitation, to a simple exponential decay law, the differential equation of the process is

$$\frac{dN}{dt} = \frac{\mu I(t)}{h\nu} - \alpha N,$$

where $I(t)$ is the energy of frequency ν incident per second, μ the absorption coefficient of the phosphor crystals, N the number of electrons in the excited level at any time t, and α the probability per unit time of a spontaneous transition

from the excited level to the ground state accompanied by the emission of radiation. For sinusoidal excitation, $I(t)$ has the form

$$I_m\left(1-\cos\frac{2\pi t}{T}\right),$$

where I_m is the maximum value of $I(t)$ occuring at $t=1/2T$, $3/2T$, etc., and T the period, so that the differential equation becomes

$$\frac{dN}{dt}=\frac{\mu I_m}{h\nu}\left(1-\cos\frac{2\pi t}{T}\right)-\alpha N.$$

This is a standard differential equation, the solution of which is

$$N=Ce^{-\alpha t}+\frac{\mu I_m}{\alpha h\nu}-\frac{\mu I_m}{\alpha h\nu}\left\{\frac{\cos\frac{2\pi t}{T}+\frac{2\pi}{\alpha T}\sin\frac{2\pi t}{T}}{1+4\pi^2/\alpha^2T^2}\right\},$$

C being the arbitrary constant of integration.

The "life" of the excited state, τ, the time taken for the number of atoms in such a state to decrease to $1/e$ of the original value in the absence of further excitation, is equal to $1/\alpha$, and, unless the excited state is accompanied by a metastable level, this can hardly be greater than 10^{-5} sec. This means that for large times $e^{-\alpha t}$ is a negligible term in the expression as α is 10^5 sec.$^{-1}$ or greater. The luminescence intensity is then described by the expression

$$\alpha N=\frac{\mu I_m}{h\nu}-\frac{\mu I_m}{h\nu}\left\{\frac{\cos\frac{2\pi t}{T}+\frac{2\pi}{\alpha T}\sin\frac{2\pi t}{T}}{1+4\pi^2/\alpha^2T^2}\right\},$$

This is a periodic function having a period equal to that of the exciting radiation. The phase shift of this function with respect to that describing the exciting radiation can be obtained by equating $\frac{d(\alpha N)}{dt}$ to zero and solving for

$$\epsilon=\frac{2\pi t_{\max}}{T}.$$

The solution is

$$\frac{2\pi}{T}\sin\epsilon-\frac{4\pi^2}{\alpha T^2}\cos\epsilon=0,$$

or

$$\tan\epsilon=\frac{2\pi}{\alpha T}.$$

The significant feature of this result is that the phase shift ϵ is independent of the intensity of the exciting radiation. The ratio of intensities at maxima and minima of αN, as obtained by substitution of this value of ϵ in the original expression, is

$$\frac{1+\frac{\alpha T}{\sqrt{\alpha^2T^2+4\pi^2}}}{1-\frac{\alpha T}{\sqrt{\alpha^2T^2+4\pi^2}}}\quad\text{or}\quad\cot^2\tfrac{1}{2}\epsilon.$$

This also is independent of the intensity I_m, and is determined only by the period of the excitation and the "life" of the excited electronic state of the activator atom in the phosphor crystal. A plot of the phase shift against the ratio of maximum to minimum emitted intensities is given in figure 1.

In the simplest case of an ionization-recombination mechanism, i.e. in the case leading, for constant excitation, to a simple bimolecular decay law, the differential equation describing the process, under the periodic conditions of excitation as before, is

$$\frac{dN_c}{dt} = \frac{\mu I_m}{h\nu}\left(1 - \cos\frac{2\pi t}{T}\right) - \beta N_c^2,$$

where N_c is the number of electrons in the conduction band at time t and β is the probability per unit time that an electron in the conduction band will encounter

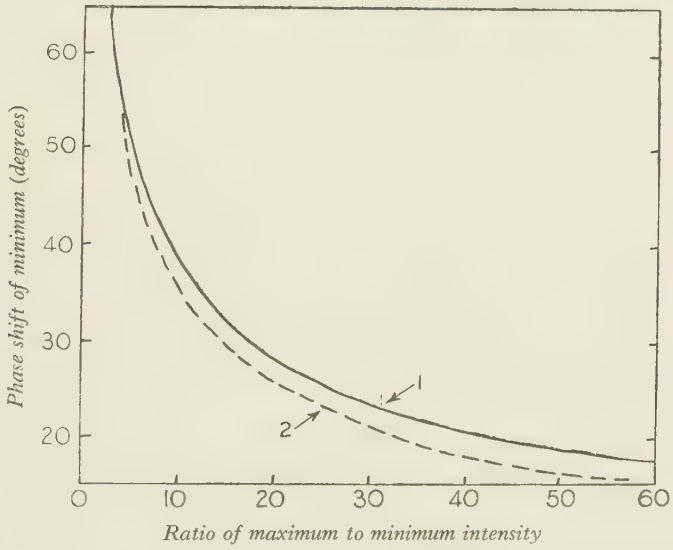

Figure 1. Theoretical relationships between phase shift of minimum luminescence intensity and ratio of maximum to minimum intensities under sinusoidal excitation.

1. Simple ionization-recombination mechanism. 2. Simple monomolecular mechanism.

a positive hole on an activator atom and recombine with it with the emission of luminescent radiation. The other symbols in the expression are defined as in the previous case.

The differential equation is not of a standard type; it is a particular form of the Riccati equation, which may be transformed into a linear form known as Hill's equation. A numerical solution has been given by Wise (in course of publication) in terms of the parameter

$$\kappa^2 = \frac{\beta\mu I_m T^2}{h\nu\pi^2},$$

and has been tabulated for values of κ^2 between 0·25 and 64. The solutions are given in the form

$$n^2 = \left(\frac{\beta N_c T}{\pi}\right)^2 = a + b'\sin\left(\frac{2\pi t}{T} + \phi'\right) + b''\sin\left(2.\frac{2\pi t}{T} + \phi''\right) + \ldots,$$

where a, b', b'',.... and ϕ', ϕ'',.... are determined by the value of κ^2. It is significant that the solution in this case depends on the maximum exciting intensity and hence, also, will the phase and the ratio of maximum to minimum emitted intensities. In table 1, the phase of the maxima and minima of βN^2 in terms of t, and the ratio of maximum to minimum luminescent intensity have been recorded for nine values of κ^2 between 0·25 and 64. These were obtained by a method of successive approximations from the numerical solutions given by Wise; the

Table 1

κ^2	Phase of minimum	Phase of maximum	Maximum luminescent intensity / Minimum luminescent intensity
$\to 0$	$\to 90°$	$\to 270°$	$\to 1$
$\frac{1}{4}$	65° 26′	242° 18′	2·47
$\frac{1}{2}$	59° 13′	233° 30′	3·40
1	50° 30′	221° 6′	4·85
2	43° 34′	211° 0′	6·87
4	38° 30′	201° 14′	10·0
8	32° 5′	194° 3′	14·2
16	26° 22′	189° 49′	21·3
32	21° 50′	187° 0′	35·4
64	17° 25′	186° 30′	63·0
$\to 8$	$\to 0$	$\to 180°$	$\to 8$

limiting values are also given. The theoretical relationships between the phase shifts at the maximum and minimum and the ratio of maximum and minimum values of βN_c^2 have been plotted in figure 1.

The form of the solution here is in marked contrast to that of the monomolecular case, and the predictions embodied in these solutions can be used to obtain certain information simply and rapidly about phosphors whose electronic processes during absorption and emission of radiation are uncertain or unknown.

§ 3. MORE COMPLEX SYSTEMS: ELECTRON TRAPS

There are several ways in which the electronic processes leading to the emission of luminescence radiation may be complicated; the most important of these are (i) the presence of more than one type of activator in the phosphor, and (ii) the existence of metastable levels, associated with abnormalities in the crystal, which are capable of trapping electrons. The differential equations describing such systems are more complex: for the case of constant excitation intensity the theoretical treatment has been developed to a stage where the mechanism proposed can be said to describe the behaviour of certain phosphors in a qualitative way (Lord, Rees and Wise, 1946). It is evident that many phosphors considered to contain only one activator type did, in fact, contain more than one, and it is maintained that the failure to fit the simple exponential and bimolecular laws to experimental observations resulted, in part at least, from such a simplifying assumption.

Neglecting the presence of electron traps for the present, we may examine the case of a phosphor, containing more than one activator type, whose absorption

and emission processes are monomolecular in nature. We note that these processes at each activator atom will be independent and that the solution of an equation of the type

$$\frac{dN_r}{dt} = \frac{\mu_r}{h\nu} I(t) - \alpha_r N_r$$

is involved for each type of centre. The total luminous intensity is then

$$\sum_{r=1}^{n} \alpha_r N_r = \sum_{r=1}^{n} \frac{I_m \mu_r}{h\nu} \left\{ 1 - \frac{\alpha_r^2 T^2}{\alpha_r^2 T^2 + 4\pi^2} \left(\cos \frac{2\pi t}{T} + \frac{2\pi}{\alpha_r T} \sin \frac{2\pi t}{T} \right) \right\}.$$

The significant features of this function, namely, the phase and the ratio of maximum to minimum intensities, are, as in the case of a single type of centre, independent of the intensity of the exciting radiation.

For the ionization-recombination mechanism the solution of a set of simultaneous differential equations is required:

$$\frac{dN_r}{dt} = \frac{\mu_r}{h\nu} I(t) - \beta_r N_r N,$$

where N_r is the number of positive holes of the rth kind in the crystal, $N = \sum_{r=1}^{n} N_r$ the total number of electrons in the conduction band at time t, β_r the probability per unit time that an electron will recombine with a positive hole of the rth kind, μ_r the absorption coefficient for centres of the rth kind. The solution of this set of equations has not yet been attempted, but it is certain that the phase shift and ratio of maximum to minimum emitted intensities will depend on the intensity of the excitation.

A complete treatment of the problem should also take into account the variation with depth of the conditions in the phosphor (Lord, Rees and Wise, 1946), but the solutions of the appropriate equations have not yet been obtained. It is not, however, anticipated that the qualitative conclusions will be modified.

The existence of "electron traps" in many crystal phosphors has been established with certainty (Randall and Wilkins, 1945 a and b), and it would be instructive to examine the way in which these traps influence the luminescence processes. For the case of constant excitation intensity in electronic processes involving the conduction band, some progress has been made towards a solution of the problem (Lord, Rees and Wise, 1946), but for periodic excitation conditions it is somewhat more complicated. However, when the period T is short, 10^{-2} sec. or less, it is expected that electron traps will play a relatively small part in the luminescence processes, and that they will not contribute appreciably to the emission, except in those systems where the electron must pass through the metastable or trapping level before reaching the level from which a transition may be made to the ground state with the emission of radiation.

Systems in which the electron is excited from atoms of the crystal matrix, followed by a migration of the positive hole to the activator atom, where the electron-positive hole recombination takes place, have not been considered here, as the case of constant excitation intensity for such phosphors has not yet been treated theoretically.

It therefore appears that it should be possible to determine the general type of luminescence mechanism followed by a phosphor by examination of its behaviour under short-period excitation. The derivation of such information is illustrated in the subsequent sections of this paper.

§4. MEASUREMENT OF TIME DEPENDENCE OF LUMINESCENCE INTENSITY

The phosphors employed in these experiments were;

(i) ZnS, activated by $7{\cdot}5 \times 10^{-3}$ % Ag, but also containing interstitial Zn and a trace of Cu ($\leqslant 1 \times 10^{-4}$ %). The phosphor therefore contained three types of activating centre (see Lord, Rees and Wise, 1946).

(ii) ZnCdS containing 18% ZnS, activated by 1×10^{-2} % Cu, and presumably containing interstitial Zn and Cd also.

It was found that the intensity of the group of lines at λ3650 from the quartz inner tube of an 80-watt MB/V high-pressure mercury-vapour lamp operated on 50 cycles varied in a substantially sinusoidal manner with time. Such a lamp constituted a convenient source of exciting radiation and was used for the experiments on the zinc and zinc-cadmium sulphide phosphors.

For recording the luminescence intensity-time relation, a cathode-ray-oscillograph method was employed. Light from the source, enclosed in a ventilated lamp-house, was passed through an 18A Wratten filter to isolate the λ3650 radiation and then on to the phosphor, coated uniformly on a thin glass plate. Owing to the proximity of the lamp, the temperature of the phosphor was about 30° C. The transmitted radiation consisted of unabsorbed λ3650 and emitted luminescence, and the former must be removed by an appropriate filter before the radiation strikes the photosensitive surface of the detector. An electron-multiplier photocell (Cinema Television, Ltd., MA 20, No. 300) was used as detector, the output from which was passed through a 1·2 megohm resistor and the p.d. appearing across this resistor applied directly to the Y-plates of a Mullard GM 3156 cathode-ray oscillograph. Synchronization is automatic if the time base and light source are both actuated by the same A.C. supply. A biasing voltage of +48 volts was required to bring the zero line to a convenient part of the oscillograph screen. The H.T. supply (*ca.* 1000 volts) for the multiplier was obtained from a rotary D.C. transformer operating on 14 v. D.C.; the generator output was smoothed by means of conventional filters before application to the multiplier potentiometer. Rigorous precautions were taken to exclude stray fields from the oscillograph.

The light filter system between the phosphor and the multiplier receiver must allow only that emitted light which it is proposed to investigate to fall on the photosensitive surface. For ZnS.Ag, whose emission is predominantly in the blue, a Wratten No. 39 (blue) filter, to remove green phosphorescence due to Cu, and a glass cell containing saturated aqueous sodium nitrate, to remove transmitted λ3650, were employed; for ZnCdS.Cu, whose emission maximum lies in the orange-red, a Wratten No. 8 (yellow) filter sufficed to remove both transmitted λ3650 and visible blue or green radiation.

Calibration of the vertical displacements in the oscillograph of the zero line and the trace with respect to exciting intensity proved necessary, since the zero

line was found to rise owing to a build-up effect in the 1·2 MΩ resistor, and the responses of the multiplier and oscillograph were not linear. Calibrations were performed with the mercury lines lying in the range of the spectral emission of the phosphor under investigation by reducing the intensity in known ratios with a calibrated neutral wedge.

The wave-forms of the luminescence for four different excitation intensities were photographed on the same plate together with the wave form of the Hg λ3650 radiation, used to fix the origin of $I(t)$. For each plate, the deflection caused by light reaching the photocell with the luminescent layer removed, but with filters in place, was photographed; however, this deflection proved to be negligible. The true zero line appropriate to each observed wave form was derived, the height above it of the maxima and minima obtained and corrected for non-linear response by means of calibration curves. All traces were recorded independently on three occasions, the traces measured from optical enlargements and the mean values taken. A typical photographic record is shown in figure 2. The absolute intensities of excitation radiation were obtained from measurements with a Moll thermopile.

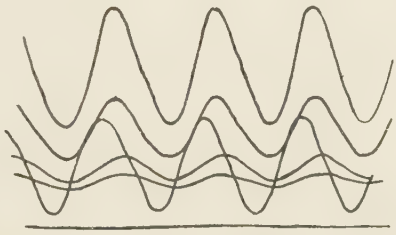

Figure 2. Reproduction of a typical oscillograph record showing phase shift.

§ 5. RESULTS AND DISCUSSION

The results of the experiments conducted are recorded in table 2 and plotted together with the theoretical curve for the region of the experimental observations, in figure 2.

Table 2

Phosphor	I (watts/cm.^{2})	Ratio of maximum to minimum intensity (mean)	Phase shift of minimum (degrees) (mean)
ZnS . Ag	$7{\cdot}0\times10^{-4}$	2·93	40·3
	3·5	2·18	43·7
	2·0	1·89	48·9
	1·0	1·53	51·7
ZnCdS . Cu	$7{\cdot}0\times10^{-4}$	9·8	22·3
	3·5	8·3	24·4
	2·0	5·8	27·8
	1·0	4·3	30·7

Owing to the presence of a certain amount of distortion of the peaks near the top of the oscillograph screen, the phases were measured only at the minima.

From the curves in figure 3 it is apparent that the qualitative behaviour of the phosphors is as predicted by the theory outlined in § 3, but the quantitative agreement is poor. This is not suprising in view of the thickness of the layer, the complex nature of the systems constituting these phosphors and the possible effect of electrons trap at the relatively low frequency of excitation used (10^2 cycles per second). However, the results demonstrate quite conclusively that the luminescence mechanism is not monomolecular in type since all points in figure 3 for one phosphor corresponding to different intensities would then be identical, and, indeed, lend support to the belief that it is, *ca.* 30° c., of an ionization-recombination type. It is not necessarily expected that the deviations from theory for the two phosphors investigated should be the same; however, it can be seen from figure 3 that both sets of points approximate to the same curve. As the

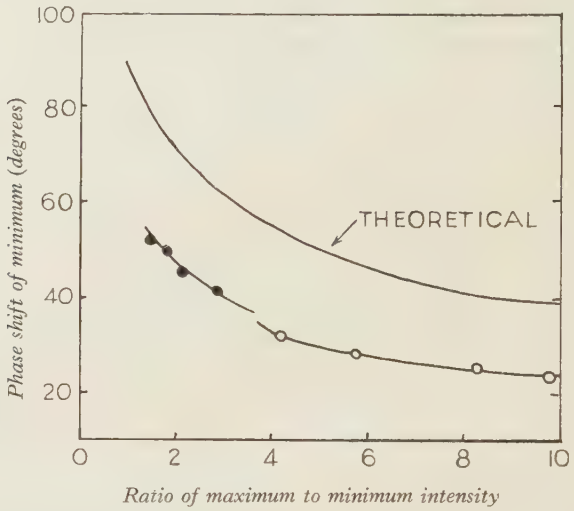

Figure 3. Comparison of theory and experiment. Theoretical curve from table 2 for ionization-recombination model. Experimental observations: ○ ZnCdS . Cu, ● ZnS . Ag.

deviations are in all probability largely due to interstitial lattice atoms (Zn or Cd), whose equilibrium concentrations at the temperature of preparation of the phosphor would be of the same order, this fact is not surprising. It is evident that all phosphors following the simple bimolecular mechanism should satisfy the theoretical relation of figure 1 irrespective of the value of the recombination coefficient, absorption coefficient, intensity or periodicity of the excitation.

It would appear that the principles of the method here discussed should be of considerable value in distinguishing between various models of the luminescence mechanism, since the interpretation of decay curves under constant excitation conditions is a matter of some difficulty and uncertainty.

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NOTE ON THE RAPID DETERMINATION OF DECAY CHARACTERISTICS OF LUMINESCENT SOLIDS

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ABSTRACT. For the provision of rapid and convenient means for the examination of luminescent solids by oscillographic methods, the possibility of producing square-wave light pulses from a gaseous discharge has been investigated. Decay curves sufficiently accurate for the estimation of half-life periods and for the study of gross effects could be obtained by interrupting the D.C. supply to the discharge by means of a small vibrator.

FOR comparative and routine purposes, a sufficient criterion of the decay of luminescence of a phosphor is its "half-life", that is, the time taken for the emitted intensity to fall to half its value after the removal of excitation. Methods less tedious and precise than the orthodox have therefore been developed to derive decay curves rapidly.

An electron-multiplier phototube together with a cathode-ray oscillograph was first used for the plotting of decay curves by de Groot (1939). His method involved the use of rotating sectors for the production of square pulses of exciting radiation; the apparatus described here provides for the production of these square pulses directly from the gaseous discharge. The specific advantage of this method lies in the absence of motor-driven moving parts, leading to a more convenient and portable equipment; it also avoids the provision of an adjustable constant-speed motor and provides simpler synchronization for the oscillograph time-base. Use was made of the same multiplier-oscillograph combination as that employed in the study of the behaviour of sulphide phosphors under periodic excitation (Lord and Rees, 1946). We were concerned here, however, with the examination of silicate materials and, therefore, used a low-pressure mercury discharge as the source of $\lambda 2537$ radiation.

Square wave-forms were produced by the following methods: (i) The use of a bright-emitter diode run under saturation conditions; (ii) the use of a thyratron in series with a saturated diode; (iii) interruption of the D.C. supply to the discharge with a mechanical vibrator. The first method did not give a completely satisfactory flat-topped wave and could not be used as such; when, however, a

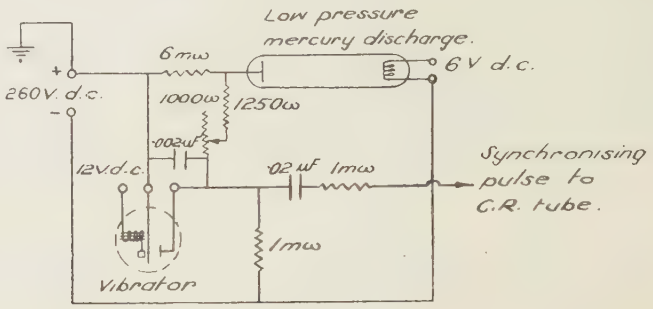

Figure 1. Circuit for production of square-wave pulses for λ 2537 radiation.

thyratron was connected in series, the wave-form was improved and variation of the pulse duration by control of the grid bias then became possible.

As the current requirements of the discharge lamp were of the order of 70 ma. for satisfactory light output, the use of a mechanical vibrator (V 6121, 12 volt) was found to be simpler. This vibrator was used in the circuit outlined in figure 1 to interrupt the 260-volt D.C. mains, which were smoothed by means of a conven-

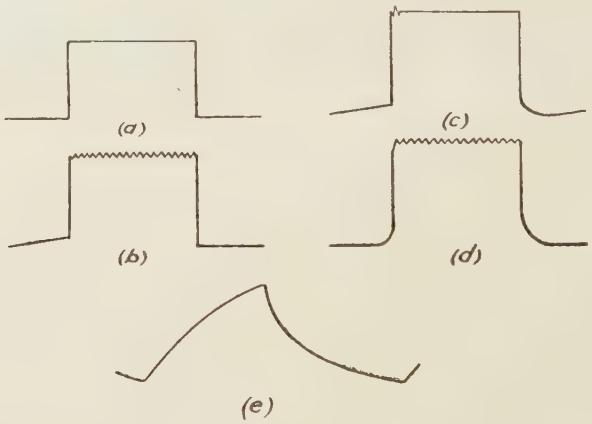

Figure 2. Wave-form characteristics of vibrator circuit.

(*a*) Discharge current; (*b*) lamp voltage; (*c*) vibrator voltage; (*d*) light output from discharge; (*e*) build-up and decay of luminescence in manganese-activated zinc silicate.

tional pi filter. A synchronizing pulse was taken to the oscillograph through a condenser and resistor as shown. The 6-megohm series resistor ensured that a constant small number of ions were available to facilitate starting up the lamp on each cycle. For currents up to 70 ma. (the lamp rating), satisfactory wave-forms were obtained, but with some build-up and decay and a small-amplitude high-frequency oscillation superimposed on the flat top. The high frequency has its origin in the discharge, as can be seen from the wave-forms in figure 2, (*a*, *b* and *c*).

The current wave-form of figure 2(*a*) was obtained by placing a series resistance of 150 ohms in the negative line and applying the voltage established across this resistor to the vertical deflector plates. Figure 2(*b*) was obtained by applying the voltage across the lamp directly to the deflector plates, using a shunt of 250,000 ohms across the lamp. The vibrator voltage wave-form, shown in figure 2(*c*), was obtained by connecting the vertical deflector plates across 50,000 ohms of the 250,000-ohm shunt; however, the discharge was suppressed in this case by removing the filament heating supply. Bounce of the contacts on make is evident at the beginning of the trace.

Curves showing the build-up and decay of luminescence, as in figure 2(*e*), are obtained by interposing the phosphor layer and appropriate filter system between the square-wave source and the photomultiplier. Half-life periods deduced from such curves are in substantial agreement with those obtained by more precise methods (Lord, Rees and Wise, 1946), provided corrections are made from calibration for the non-linearity of the multiplier-cathode-ray oscillograph system.

The method may be extended to a wide range of pulse durations and repetition rates by substituting vibrators of different frequencies and times of contact. The vibrators do not perform satisfactorily if maximum current is drawn for long periods; the wave form deteriorates if the unit becomes heated.

Although neither this method nor that of de Groot can be used to obtain decay curves sufficiently precise to give useful data about the electronic processes accompanying luminescence, this method is quite adequate for the study of gross effects and for the rapid comparison of phosphors.

ACKNOWLEDGMENTS

The authors wish to thank Mr. J. A. M. van Moll and the Directors of Philips' Lamps, Ltd., for permission to publish this note.

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BAND SYSTEMS IN THE SPECTRUM OF NITROGEN

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ABSTRACT. Several new band systems of N_2 which have been reported by us independently during the war are compared. Several systems have been obtained by both of us, and agreement is generally satisfactory. There appear to be eight progressions of bands in the ultra-violet which are due to transitions to the $a\,^1\Pi$ state. Differing vibrational analyses for the Fifth Positive (Van der Ziel) system are discussed, and it is shown that Gaydon's interpretation is to be preferred, while for Kaplan's Second system (Herman's α) Herman's vibrational analysis is more satisfactory. Bands in the green region are compared.

§ 1. INTRODUCTION

During the war we have independently investigated band spectra obtained in emission from nitrogen in discharge tubes and have each found several new systems (Gaydon, 1944 a, 1944 b; Gaydon and Worley, 1944; Herman, 1945). On the whole our results are in fairly good agreement, but there are some ways in which our interpretations of the observations differ. The names which we have applied to the various new systems are naturally quite different. Thus a joint publication seems desirable in order to assist others to sort out the numerous systems attributed to N_2 which have now been found and, as far as possible, to clear up some of the differing interpretations of the observations.

Generally speaking, Herman has developed rather more intense sources, and has thus been able to obtain some of the band systems more completely, so enabling the more extensive and reliable vibrational analyses. Gaydon, on the other hand, has used spectrographs of larger dispersion and has concentrated more on the rotational structure of the bands; data obtained from rotational analyses of a number of bands have led to definite conclusions on the nature of the electronic levels.

The characteristics of the discharge-tube sources used for exciting the spectra have been rather different, and it is not surprising to find that while some of the new band systems have been observed by both of us, other systems have been reported by only one of us.

§ 2. BAND SYSTEMS INVOLVING TRANSITIONS TO THE $a\,^1\Pi$ STATE

Many of the new systems consist of single progressions of bands showing the same vibrational intervals as those of the $a\,^1\Pi$ state, which is the upper level of the main Lyman system of N_2. This level was formerly believed to be $^1\Pi_u$ but Herzberg (1946) has recently suggested that it may be $^1\Pi_g$. These band systems

thus consist of transitions from a number of highly excited levels to the $a\,^1\Pi$ level, this being confirmed in several cases by the rotational analyses. In table 1 we have set out brief particulars of these systems, with the names which we have given to them in out separate papers.

Table 1. Band systems with $a\,^1\Pi$ as final level

ν_0	Approx. λ (0,0)	(0,1)	(0,2)	Gaydon's designation	Herman's designation	Type of upper electronic state
35371	2827	2967	3119	P	ζ	$^1\Sigma$
35761*	2795	2932	3079	—	δ, $v'=0$	
36394	2746	2878	3020	Q	δ, $v'=1$	$^1\Pi$
37417	2672	2796	—	R	—	$^1\Sigma$
41705	2397	2497	2603	S	—	$^1\Sigma$
42373*	2359	2455	2558	—	θ	
43818	2281	2371	—	T	ϵ	$^1\Sigma$
46611*	—	2224	2308	—	η	

* These values are derived from head measurements; other values of ν_0 are for band origins.

If the $a\,^1\Pi$ state is $^1\Pi_u$, then the $^1\Sigma$ states will be $^1\Sigma_g^-$, as given in Gaydon's paper, and the upper $^1\Pi$ state will be $^1\Pi_g$. If, however, the $a\,^1\Pi$ state is $^1\Pi_g$, then the $^1\Sigma$ states will all be $^1\Sigma^+$ and the upper $^1\Pi$ will be $^1\Pi_u$.

For the system with $\nu_0=35371$ our analyses agree; these bands appear more strongly on Gaydon's plates.

The two progressions with ν_0 35761 and 36394 were tentatively grouped together as one system by Herman. However, since the first progression is not present on Gaydon's plates, it now seems more probable that the two progressions belong to separate systems. The value of ω_e, about 640 cm.$^{-1}$, obtained by combining the two progressions, also seems rather small for a state with a B value of about 1·36. Herman obtained four bands in each progression. Gaydon obtained only the three strongest bands of the ν_0 36394 progression, but achieved rotational analyses of two of these. Janin (1943) has independently observed bands of both of these progressions, his measurements agreeing with ours.

The 37417 progression was only reported by Gaydon, who made a rotational analysis of the (0,0) band and also found the (0,1) band, which, it now appears, very nearly coincides with the (0,0) band of the 35761 progression. The (0,0) band at λ2672 is, however, present on Herman's plates.

The $\nu_0=41705$ progression again was only reported by Gaydon, but some of these bands are also present on Herman's plates.

The 42373 progression was reported only by Herman, but there is some weak structure in the region of these bands on Gaydon's plates as well.

The 43818 progression has been obtained by both of us. Gaydon discussed the possibility that this might be the $v'=1$ progression of the system with $\nu_0=41705$, but decided the evidence was not sufficiently conclusive.

The progression with ν_0 at 46611 was reported only by Herman, but the not (0,1) and (0,2) bands at λ2224 and 2308 are present on Gaydon's plates, although higher members of the progression are absent or masked.

Thus there is evidence for the existence of eight independent systems of N_2 in the ultra-violet all having the common final level $a\,^1\Pi$. Most of these systems have been observed by both of us, although our original publications only gave three systems in common.

The close numerical coincidence between the energies of the upper levels of Herman's ϵ and ζ systems with those of the levels h and c obtained from absorption measurements by Watson and Koontz and by Birge and Hopfield led her to think that these levels, h and c, were the upper levels of the ϵ and ζ systems, thus indicating a violation of the selection rule that the symmetry u or g must change. Herzberg (1946) has now suggested that the $a\,^1\Pi$ state is of g symmetry and that the forbidden transition is the Lyman system itself, $a\,^1\Pi_g \rightleftharpoons X\,^1\Sigma_g^+$. This accounts for the great path length of gas required to produce the Lyman bands in absorption. It brings the upper level of the 43818 progression into coincidence with the level h, as suggested by Herman. The upper level of the 36394 progression would then also fit with Worley's level m, but the upper level of the 35371 progression could only be identified with the level c by assuming that the electronic type of this state c is $^1\Sigma_u^+$ and not $^1\Pi_u$, as formerly supposed; the B values agree satisfactorily. There is, therefore, much to be said for this suggestion by Herzberg, but it should be pointed out that it does not explain why strong absorption bands to the upper levels of our other progressions have not been reported.

§3. THE FIFTH POSITIVE OR VAN DER ZIEL'S SYSTEM AND KAPLAN'S SYSTEMS

Three other band systems of N_2 with related energy levels have also been studied by both of us. Our designations for these systems are given in table 2.

Van der Ziel originally reported two progressions of bands separated by an interval of 1827 $cm.^{-1}$, and he assumed that these were the $v' = 0$ and 1 progressions of a system. The intensity distribution in this system on this assumption was obviously unsatisfactory. Gaydon obtained these two progressions on his plates and also found an additional progression separated by 1869 $cm.^{-1}$ from the

Table 2

Gaydon's designation	Herman's designation
Fifth Positive (van der Ziel)	Van der Ziel
Kaplan's First System	β
Kaplan's Second System	α

first of the older progressions. By treating the new progression as the $v' = 0$ and the original progressions as $v' = 1$ and 2, a satisfactory intensity distribution, in conformity with the Franck-Condon principle, was obtained. Rotational analysis of one band of the new progression proved that it belonged to the same system as the older progressions.

Van der Ziel's rotational analysis had shown that the bands were due to a transition between two Σ states, and Herman made an attempt to identify the upper state with that of a system of bands observed by Collins and Price in absorption, it being assumed that van der Ziel's two progressions arose from levels with

$v = 1$ and 3 of this state. By assuming preferential excitation to these levels a reasonable intensity distribution could be obtained. However, it is now found that Gaydon's additional progression is also present on Herman's plates, and as this cannot be fitted into Herman's scheme it seems that Gaydon's interpretation is to be preferred. Moreover, Gaydon points out that Herman's scheme would be inconsistent with the strong degradation of the bands to shorter wave-lengths, and that it is now known that the bands are due to a transition between two $^1\Sigma$ levels, whereas the upper level of Collins and Price's system is believed to be a triplet.

Kaplan originally reported a system with (0,0) band at 2153 A. and gave measures of six bands to the nearest angstrom, and suggested that they might involve a transition to the same final level as that involved in van der Ziel's system. Gaydon observed only three bands clearly, with a suggestion of a fourth, but was able to make an analysis of the (0,2) band, showing that the transition was $^1\Pi \rightarrow {}^1\Sigma$ and obtaining the value of B', and confirming that B'' had the same value as for the Fifth Positive system. Herman obtained a large number of bands (seven in all) and was able to show that $\omega_{\frac{1}{2}}' = 1708\,\mathrm{cm.}^{-1}$.

Kaplan's second system of bands, which he also believed to have the same final level as van der Ziel's, has been observed by us. Gaydon only obtained the bands weakly and was unable to make a rotational analysis, and said that the vibrational intervals agreed as well as could be expected with those of the lower state of the Fifth Positive system. Herman, however, obtained this system more strongly, and recorded eleven bands, including new bands at $\lambda\lambda$ 2354 and 2263, which are also subsequently found to be present quite clearly on Gaydon's plates and which prove that the transition does not involve the lower state of van der Ziel's system, as had been thought by Kaplan and Gaydon. The vibrational energy interval for the upper state is about $1709\,\mathrm{cm.}^{-1}$, which suggests that this system has the same initial $^1\Pi$ level as Kaplan's first system.

§ 4. OTHER SYSTEMS

For two other systems our respective designations are given in table 3.

Herman's γ system consists of two fairly long progressions, and the analysis shows that the transition is to the A $^3\Sigma$ level of N_2. Bands of this system are not present on Gaydon's plates. Kaplan had previously observed a number of these

Table 3

Gaydon's designation	Herman's designation
(Kaplan's third system)	γ
Green system	χ

bands and had suggested that some of them formed a progression involving the lower level of van der Ziel's system. This is now shown to be incorrect.

A system of rather complex bands in the green region was observed by Gaydon, who arranged seven bands into a tentative vibrational scheme. Herman also reported three bands at $\lambda\lambda$ 5594, 5330 and 5092 under small dispersion and referred to them as the χ system. More recently Herman has obtained this system with greater intensity, and revised measurements of five bands show clearly that this χ system is identical with Gaydon's Green system.

Mutual comparison of our results has led to the identification of a number of the unassigned bands on our plates, but there are still a number of weak bands, probably due to N_2, especially at the far end of the quartz ultra-violet region, which have not been analysed.

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DISCUSSION

Dr. S. P. SINHA. I would like to know the experimental conditions under which the new systems of nitrogen bands discovered by Dr. A. G. Gaydon were developed and if it was possible to explain their appearance in the light of the conditions maintained during the experiment.

AUTHOR'S reply. The experimental conditions for exciting the new band systems were fully described in earlier papers (Gaydon, 1944 a, 1944 b) and the conditions of excitation were discussed. Briefly, the ultra-violet systems were obtained by a condensed induction coil discharge through N_2 in a simple Pyrex tube, while the Green system required a Tesla discharge at relatively high gas pressure.

A SYSTEM OF TRANSFER COEFFICIENTS FOR USE IN THE DESIGN OF LENS SYSTEMS: VI. THE CHROMATIC VARIATION OF THE TANGENTIAL ABERRATIONS

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University of Tasmania

MS. received 18 *July* 1945

ABSTRACT. Using those differential transfer coefficients of a lens system which specify the rate of change of the tangential aberrations with the refractive indices of the component singlet lenses of the system, a simple method is available for the rapid calculation of the chromatic variation of these aberrations. In a similar manner, other transfer coefficients (the chromatic coefficients of the system) provide a rapid means of obtaining a complete analysis of the secondary spectrum of the system at any zone in the case of the axial aberration, and at any obliquity of the transverse aberration. Examples of an actual system are given.

§ 1. INTRODUCTION

THE aberrations of an optical system are determined normally for incident light of some mean wave-length in the spectral range over which the system is expected to give satisfactory images. The full significance of the chromatic aberration of the system is really apparent only when, in addition to the usual analysis, the variation of the aberrations with the wave-length of the incident light

has been examined. Hitherto the analysis of the dependence of the aberrations on the wave-length of the incident light has required repeated ray-tracings in different colours. It is of interest, therefore, to examine this matter in the light of the properties of the transfer coefficients which have been developed in the earlier papers of this series.

Suppose that the aberrations of the system for incident light of some mean wave-length, d, have been obtained by a ray-trace and the general transfer coefficients have been calculated from the trace. If the mean refractive indices of the glasses of the components a, b, ... h, ... of the system are changed by amounts δN_a, δN_b, ... δN_h, ..., the change in the value of the aberration A'_d which may be any of the tangential aberrations of the system, resulting from the glass changes is given by

$$\delta A'_d = \Sigma \frac{\partial A'_d}{\partial N_h} \delta N_h.$$

If, however, instead of considering glass changes we consider a change in the wave-length of the light which is incident on the system from the object, then the value of the aberration, A', for light of wave-length, r, will be given by

$$A'_r = A'_d + \Sigma \frac{\partial A'_d}{\partial N_h} (N_r - N_d)_h.$$

Hence, using the computed values of the transfer coefficients, $\partial A' / \partial N_h$, for the various aberrations, we can investigate the chromatic variations of the aberrations quite quickly.

§ 2. A TYPICAL EXAMPLE

As an example we consider a rough design of a wide-angle photographic objective of the Ross Xpres type. In table 1 the computed values of the transfer coefficients of the aberrations with respect to the refractive indices of the components are shown, the oblique aberrations being calculated from a pencil of about 22° obliquity.

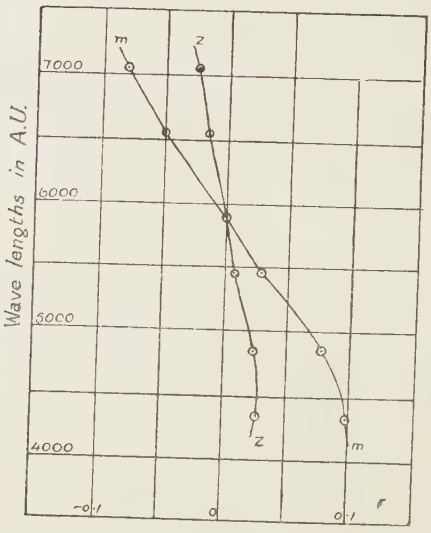
Table 1

	LA'_T	LA_z	X'_T	Coma$'_T$	Dist$'$
$\partial/\partial N_a$	150·13	52·302	−100·48	0·891	7·780
$\partial/\partial N_b$	−140·51	−49·202	115·59	0·167	−7·029
$\partial/\partial N_c$	7·11	1·960	− 23·54	−1·887	1·382
$\partial/\partial N_d$	12·53	5·220	− 8·48	1·195	−1·322
$\partial/\partial N_e$	−115·64	−42·750	47·16	−0·651	5·529
$\partial/\partial N_f$	117·16	43·000	− 35·34	−0·240	−6·180

The glass selected for components a and f was DBC 615553, for components b and e LF 549467, and for components c and d HC 519604. The dispersions of these glasses are given in table 2. The ray-trace of the system having been carried out for sodium light, we calculate next the aberration differences, $A' - A'_D$, for wave-lengths from 7065 A. to 4341 A. Since the glasses in the front and rear halves of the lens are similar, we may combine the coefficients for the components which have similar glasses, as, in the evaluation of the differences by the equation given above, these coefficients will each be multiplied by the same dispersion values. In table 3 the full calculation is set out for the case of the marginal spherical

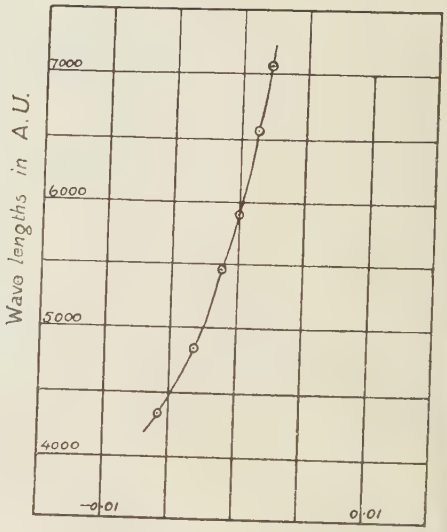
aberration, and finally, in table 4 the chromatic differences for all the aberrations are collected, the detailed computations being omitted.

The accompanying graphs in figures 1 to 3 show the variation of the aberrations plotted against the wave-length, and thus completely summarize the analysis of the



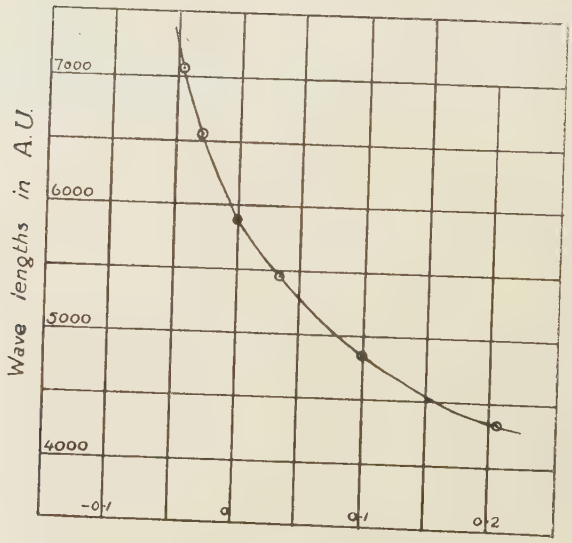
$LA' - LA'_D$

Figure 1



$\text{Coma}'_T - \text{Coma}'_{TD}$ (22° pencil)

Figure 2.



$X'_T - X'_{TD}$ (22° pencil)

Figure 3.

dependence of the general correction state of the system on the wave-length of the incident light. As a check on the general validity of this analysis, the differences of the spherical aberration for C and D light, and for F and D light, were determined by complete ray traces and compared with the values computed by the present

method. The results are given in table 5, the discrepancies being little greater than the uncertainty of the last figure of the traces.

Table 2

	DBC	LF	HC
$N_b - N_D$	−0·00517	−0·00539	−0·00406
$N_C - N_D$	−0·00328	−0·00342	−0·00256
$N_e - N_D$	0·00274	0·00291	0·00211
$N_F - N_D$	0·00785	0·00835	0·00603
$N_{G'} - N_D$	0·01422	0·01529	0·01089

Table 3

Comp.	Coeff.	Contrbn. b–D	Contrbn. C–D	Contrbn. e–D	Contrbn. F–D	Contrbn. G′–D
a, f	267·29	−1·3819	−0·8767	0·7324	2·0982	3·8008
b, e	−256·15	1·3806	0·8760	−0·7454	−2·1388	−3·9164
c, d	19·64	−0·0798	−0·0503	0·0415	0·1185	0·2140
$LA' - LA'_D =$		−0·0811	−0·0510	0·0285	0·0779	0·0984

Table 4

Range	LA'_m	LA_z'	X'_T	Coma'_T	Dist′
b–D	−0·0811	−0·0263	−0·0450	0·00205	−0·00043
C–D	−0·0510	−0·0165	−0·0291	0·00129	−0·00027
e–D	0·0285	0·0087	0·0339	−0·00109	0·00015
F–D	0·0779	0·0236	0·0996	−0·00310	0·00040
G′–D	0·0984	0·0275	0·2082	−0·00568	0·00047

Table 5

	Marginal		Zonal	
	Trace	Coeffs.	Trace	Coeffs.
$LA'_C - LA'_D$	−0·066	−0·051	−0·018	−0·0165
$LA'_F - LA'_D$	0·071	0·0779	0·022	0·0236

§ 3. THE ANALYSIS OF THE SECONDARY SPECTRUM

To complete the discussion of the chromatic variation of the aberrations, the matter of the secondary spectrum should be considered. It has been shown previously that the final intersection length, L'_r, for an axial ray of some colour r is related to the corresponding quantity, L'_d, for the traced ray for some mean wave-length, d, by

$$L'_r = L'_d + \Sigma \frac{\partial L'_d}{\partial N_h}(N_r - N_d)_h,$$

and for a principal ray of an oblique pencil

$$H'_r = H_d + \Sigma \frac{\partial H'_d}{\partial N_h}(N_r - N_d)_h,$$

where H' measures the height at which the principal ray intersects a fixed plane, conveniently the paraxial image plane. These transfer coefficients, $\partial L'/\partial N_h$, and $\partial H'/\partial N_h$, which may be termed appropriately the *chromatic coefficients* of the system, provide a means of analysing the importance of the secondary spectrum as it affects the axial and transverse chromatic aberrations of the system. It will also be obvious that they furnish a simple means of determining the effect on the colour correction of changes in the glasses of the system provided these are not too drastic.

The continuation of our example will afford a clear insight into the use of these chromatic coefficients. In table 6 are collected the computed values of the chromatic coefficients for the marginal, zonal, and paraxial rays of the axial pencil, and the principal ray of an oblique pencil at 22°.

Table 6

Comp.	$\partial L'_m/\partial N$	$\partial L'_z/\partial N$	$\partial l'/\partial N$	$\partial H'/\partial N$
a	−737·99	−640·16	−587·86	−39·917
b	768·53	677·23	628·03	30·309
c	−162·67	−157·51	−155·56	− 6·300
d	−127·07	−119·76	−114·53	8·881
e	574·99	502·11	459·53	−30·148
f	−521·13	−446·98	−403·98	38·215

Table 7

Comp.	Chr. Coeff.	Contrbn. b–D	Contrbn. C–D	Contrbn. e–D	Contrbn. –FD	Contrbn. G′–D
a, f	−1087·14	5·6205	3·5658	−2·9788	−8·5340	−15·4591
b, e	1179·33	−6·3566	−4·0333	3·4319	9·8478	18·0320
c, d	− 277·27	1·1257	0·7098	−0·5850	−1·6719	− 3·0195
	$L_z'-L'_{zD}$	0·3896	0·2423	−0·1319	−0·3585	− 0·4466

In table 7 the contributions made by the pairs of components of similar glass to the zonal chromatic aberration, $L'_z - L'_{zD}$, are computed for the various spectral ranges, the last line of the table giving the final value of the aberration. Omitting the corresponding detailed computation for the other three rays under consider-

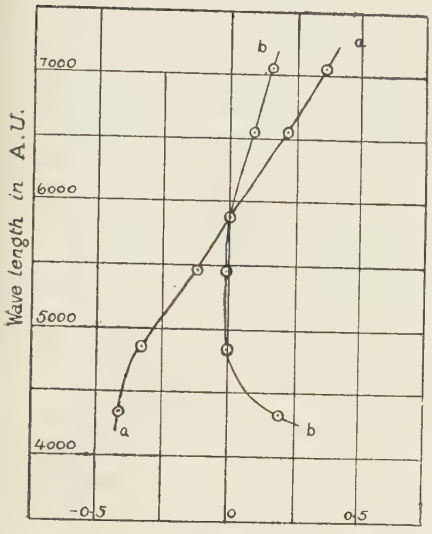
Table 8

Aberration	b–D	C–D	e–D	F–D	G′–D
$L'_m-L'_{mD}$	0·4443	0·2767	−0·1518	−0·4128	−0·5176
$L'_z-L'_{zD}$	0·3896	0·2423	−0·1319	−0·3585	−0·4460
$l'-l'_D$	0·3634	0·2258	−0·1232	−0·3349	−0·4192
$H'-H'_D$	−0·00255	−0·00158	0·00126	0·00354	0·00638

ation, we collect in table 8 the values of the aberrations calculated for each ray. This provides a sampling of the axial colour correction in the paraxial, zonal, and marginal regions, and a single sampling of the transverse colour at one obliquity.

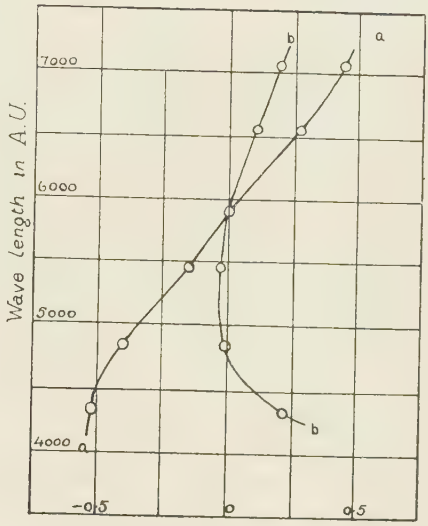
The curves marked *aa* in figures 4 to 7 show these aberration values plotted against the wave-length, and hence summarize completely the condition of the

system as regards its secondary spectrum. The axial chromatic aberration is very undesirable and calls for attention. A study of the chromatic coefficients and the contributions to the aberrations shows that an improvement should be effected by increasing the V-number of the DBC components and also the HC components.



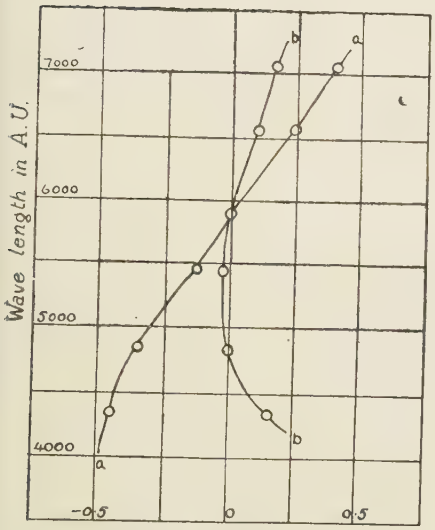
$L'_m - L'_{mD}$

Figure 4.



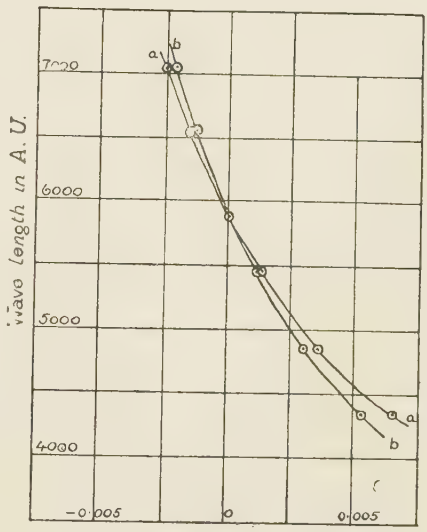
$L_z' - L'_{zD}$

Figure 5.



$l' - l'_D$

Figure 6.



$H' - H'_D$

Figure 7.

By way of example we consider the result of changing to a DBC of the type 613568, and also substitute a BSC of the type 517641 for the HC of components c and d. Ten minutes' computation with the new dispersion values in place of the former ones gives a new set of aberration values. Omitting the details, these are collected

in table 9, from which the curves *bb* in figures 4 to 7 have been drawn. These curves show a very great improvement in the axial colour correction and a slight improvement in the already good transverse aberration. This example affords clear evidence of the general usefulness of these chromatic coefficients of the system.

Table 9

Aberration	b–D	C–D	e–D	F–D	F′–D
$L'_m - L'_{mD}$	0·1888	0·1073	−0·0250	−0·0160	0·2198
$L_z' - L'_{zD}$	0·1627	0·0920	−0·0199	−0·0059	0·2097
$l' - l'_D$	0·1525	0·0861	−0·0196	−0·0069	0·1920
$H' - H'_D$	−0·00221	−0·00136	0·00116	0·00298	0·00523

As a final check on the method we investigate the reliability of the curves *aa* as an analysis of the secondary spectrum. Ray-tracings were made for the three axial rays in colours C and F and the final intersection values for each compared with the values deduced by using the chromatic coefficients.

Table 10

Wave-length	Marginal		Zonal		Paraxial	
	Trace	Coeffs.	Trace	Coeffs.	Trace	Coeffs.
D	108·445	—	106·522	—	106·525	—
C	108·734	108·721	106·766	106·764	106·748	106·751
F	108·030	108·032	106·159	106·163	106·181	106·190

The first line in table 10 gives the intersection lengths for D light for the three rays in the original trace. From these values and the aberration quantities in table 8, the predicted values of the intersection lengths of the rays for C and F light are obtained and entered in the table. The values obtained from the complete traces are entered alongside the predicted values, from which it is seen that the agreement is within the limits of uncertanty of the traced values.

MENDELSSOHN'S *z*-PARTICLES AND THE BOSE-EINSTEIN STATISTICS

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ABSTRACT. Bose-Einstein statistics are applied to Mendelssohn's *z*-particles and shown to give a satisfactory qualitative explanation of the phenomena of liquid helium.

§1. INTRODUCTION

DR. MENDELSSOHN has recently propounded the concept of a *z*-state in which *z*-particles (either atoms or electrons) occupy a set of quantum states lower than the ordinary thermal states and separated from them by a finite energy gap. The *z*-particles form an aggregate in momentum space with zero

thermal energy, and transport phenomena are caused by a diffusion of z-particles under their own zero-point momentum.

As Mendelssohn remarks, this concept should aid the development of a satisfactory theory both of liquid helium and of superconductivity. However, he appears to doubt whether the Bose-Einstein statistics are capable of handling the situation for helium atoms, and favours the possibility that some kind of compromise statistics may be found that will work alike for electrons and helium atoms in the z-state. The present writer believes this to be essentially erroneous, if only because of the fact that the two types of statistics arise from mutually inconsistent symmetry conditions on the wave functions representing the particles ; and it is difficult to see what better compromise can be found than the classical statistics, which obviously do not apply to either case.

The object of the present note is to show that, in fact, Mendelssohn's concept can be harmonized with the Bose-Einstein statistics when the latter is suitably applied to the model proposed by Mendelssohn. Previous attempts to apply the Bose-Einstein statistics to the liquid helium problem have all neglected the possibility that two competing sets of states exist, and that a statistical equilibrium has to be set up between them. When this equilibrium is set up, the Bose-Einstein statistics give a very satisfactory explanation of the phenomena in liquid helium.

§ 2. STATISTICAL CHARACTERISTICS OF THE Z-STATE

The z-particles with zero, or at any rate with less than normal, thermal energy must be treated statistically as having fewer degrees of freedom than the three normally characterizing the thermal states. If it is provisionally assumed that they have but two degrees of freedom, we at once arrive at a suggestion of a mechanism whereby this reduction may in fact occur : because the z-state is certainly associated very intimately with the *surface* layers of liquid helium, especially in the frictionless transport of helium atoms in films.

Without, however, attempting to specify the mechanism, let us accept the hypothesis that the z-particles have two degrees of freedom only, and that as a crude approximation we may regard the energy states of the z-particles as determined by Schrödinger waves in resonance with the surface of the assembly. This concept agrees qualitatively with Mendelssohn's requirement that the z-particles shall have a higher degree of order than the thermal states, and form a separate phase in momentum space alone. It is further necessary to assume, so as to coincide with the Mendelssohn model, that the energy zero of the surface states is at a finite level below the lowest thermal states, so that there will be no momentum transfer from the surface or z-states at sufficiently low temperature.

The energies of the thermal states are

$$\epsilon_t = \boldsymbol{h}^2(s^2+t^2+u^2)/8m_tV^{2/3}, \quad s, t, u = 1, 2, \ldots \qquad \ldots\ldots(1)$$

and of the z-states

$$\epsilon_z = -\chi + \boldsymbol{h}^2(s^2+t^2)/8m_zA, \qquad s, t = 1, 2, \ldots \qquad \ldots\ldots(2)$$

where A is a crude approximation for the effective area of the assembly, and $-\chi$ is the Mendelssohn energy gap : by hypothesis χ is positive.

§ 3. STATISTICAL EQUILIBRIUM BETWEEN *Z*-STATES AND THERMAL STATES

If there are in all N particles distributed among the two sets of states, then the Bose-Einstein statistics will lead to the equilibrium given by

$$N = N_t + N_z, \qquad \ldots\ldots(3)$$

where
$$N_t = \sum_{stu} \sum_{j=1}^{\infty} \lambda^j \exp\left[- \frac{j\boldsymbol{h}^2(s^2+t^2+u^2)}{8m_t\boldsymbol{k}TV^{2/3}} \right] \qquad \ldots\ldots(4)$$

and
$$N_z = \sum_{st} \sum_{j=1}^{\infty} \lambda^j \exp\left[j\chi/\boldsymbol{k}T - \frac{j\boldsymbol{h}^2(s^2+t^2)}{8m_z\boldsymbol{k}TA} \right] \qquad \ldots\ldots(5)$$

are the mean numbers in the thermal and z-states respectively.

Write
$$\lambda^* = \lambda e^{\chi/\boldsymbol{k}T} > \lambda\,, \qquad \ldots\ldots(6)$$

so that
$$N_z = \sum_{st} \sum_{j=1}^{\infty} \lambda^{*j} \exp\left[- \frac{j\boldsymbol{h}^2(s^2+t^2)}{8m_z\boldsymbol{k}TA} \right]. \qquad \ldots\ldots(7)$$

The expression (4) was studied by Fowler and Jones. It was summed by means of a theta-function transformation and shown to approach the following limit for large N:

$$N_t = (2\pi m_t\boldsymbol{k}T)^{3/2} V\boldsymbol{h}^{-3} \sum_{j=1}^{\infty} \lambda^j/j^{3/2}, \qquad \ldots\ldots(8)$$

provided
$$T > T_0, \qquad \lambda < 1\,,$$

where
$$2\pi m_t\boldsymbol{k}T_0 \sum_{j=1}^{\infty} 1/j^{3/2} = (N_t/V)^{3/2}\boldsymbol{h}^2. \qquad \ldots\ldots(9)$$

The corresponding summation of (5) leads to

$$N_z = 2\pi m_z\boldsymbol{k}TA\boldsymbol{h}^{-2} \sum_{j=1}^{\infty} \lambda^{*j}/j. \qquad \ldots\ldots(10)$$

So long as $\lambda^* < 1$ the number $N_z \ll N_t$ and can be neglected. The parameter λ is determined then by the equation

$$N = (2\pi m_t\boldsymbol{k}T)^{3/2} V\boldsymbol{h}^{-3} \sum_{j=1}^{\infty} \lambda^j/j^{3/2}.$$

As T falls, λ increases, and as soon as

$$\lambda \to e^{\chi/\boldsymbol{k}T} < 1, \qquad \lambda^* \to 1,$$

the series for N_z will begin to diverge, so that N_z can no longer be neglected. There is thus a critical temperature, where

$$N = (2\pi m_t\boldsymbol{k}T_c)^{3/2} V\boldsymbol{h}^{-3} \sum_{j=1}^{\infty} e^{-j\chi/\boldsymbol{k}T_c}/j^{3/2}. \qquad \ldots\ldots(11)$$

Below this temperature λ remains almost at the value $e^{-\chi/\boldsymbol{k}T}$, so (3) becomes

$T \leqslant T_c$

$$N = (2\pi m_t\boldsymbol{k}T)^{3/2} V\boldsymbol{h}^{-3} \sum_{j=1}^{\infty} e^{-j\chi/\boldsymbol{k}T}/j^{3/2} + 2\pi m_z\boldsymbol{k}TA\boldsymbol{h}^{-2} \sum_{j=1}^{\infty} \lambda^{*j}/j. \qquad \ldots\ldots(12)$$

The population of the z-states is thus negligible for $T > T_c$, and below T_c it is given by

$$N_z = N\left[1 - \left(\frac{T}{T_c}\right)^{3/2} \left\{ \frac{\sum_{j=1}^{\infty} e^{-j\chi/\boldsymbol{k}T}/j^{3/2}}{\sum_{j=1}^{\infty} e^{-j\chi/\boldsymbol{k}T_c}/j^{3/2}} \right\} \right]. \qquad \ldots\ldots(13)$$

As Mendelssohn remarks, the variation of N_z is more rapid than would be given by

$$N_z = N\,[1 - (T/T_c)^{3/2}],$$

and (13) will in fact give good agreement with experimental data provided χ is suitably chosen.

§ 4. CONCLUDING REMARKS

The present writer, with Dr. Hsu, formerly (1941) worked out a theory of liquid helium based upon the idea of surface resonance, and obtained remarkable agreement with observational data on specific heat and thermomechanical phenomena. The chief weakness of that theory was that no definite explanation was found for the frictionless nature of the film transport, although indeed the concept of diffusion by zero-point momentum (there called intrinsic pressure) was suggested.

Mendelssohn has now pointed out that a finite energy gap between the z-states and the thermal states will prevent frictional transfer, and by incorporating this idea, as in the present paper, the Bose-Einstein statistics can certainly be made to give a satisfactory qualitative explanation of the phenomena of liquid helium.

There are two important details which the theory cannot handle in its present form: First, the critical temperature is in fact hardly more than half that predicted by (11), and second, the critical temperature follows the λ-line in its dependence on external pressure.

But it is almost obvious that the discrepancy here is due to molecular forces, and the fact that the energy states (1) and (2) are derived on the assumption that the container is free from potential irregularities. It is clearly quite hopeless to improve matters in this respect until a better theory of liquids is available, and the Bose-Einstein statistics applied to the helium system as a liquid rather than a gas.

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DISCUSSION

on paper by S. BAXTER entitled "The thermal conductivity of metals" (*Proc. Phys. Soc.*, **58**, 105).

Mr. J. B. CARNE. I am prompted to draw attention to the account of the early work of Sir Benjamin Thomson, which was presented to the Royal Society in 1786 ***et seq.***

From a systematic series of beautifully simple experiments, Sir Benjamin deduced not only the now generally accepted explanation of the mechanism of heat transfer by convection, but also the existence of a layer of stagnant air which acts as a barrier to the transmission of heat from a hot surface, the postulate invariably attributed to Langmuir.

Reference to the papers will also show that Thomson found the same variation of conductivity with density for very low bulk densities of air-fibre mixtures as recorded by Baxter. It is interesting to note also that Sir Benjamin Thomson also enquired into the relation between water content of air-fibre mixtures and their conductivity.

THE CONSTRUCTION AND USE OF A "FLY'S EYE" FOR ASSISTING X-RAY STRUCTURE ANALYSIS

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MS. received 28 *December* 1945. *Read* 15 *March* 1946.

ABSTRACT. The "fly's eye" is the name given to a multiple camera for producing two-dimensional diffraction gratings consisting of repeated patterns representing projections of crystal structures. This paper describes the construction of a fly's eye using lenses embossed on Perspex; also the necessary adjustments, the method of photographing a structure, and the apparatus for viewing the spectra produced by the two-dimensional gratings so obtained.

§ 1. INTRODUCTION

The purpose of the "fly's eye" is to produce on a photographic plate a regularly repeated pattern representing the projection along some crystallographic direction of a crystal structure. This behaves as a two-dimensional diffraction grating and gives spectra whose intensities are proportional to the intensities of X-ray reflections in a zone corresponding to the direction of projection. The use of the fly's eye thus obviates the calculation of structure amplitudes of a proposed structure in the trial-and-error stages of structure determination. The purpose of the present paper is to describe the construction and setting-up of a fly's eye; the theory of its use is to be given in subsequent papers.

§ 2. DEVELOPMENT OF THE INSTRUMENT

The first method by which a repeated pattern was produced (Bragg, 1944) was by means of a pin-hole camera with an array of pin-holes lying on a square lattice. This was made by photographing an array of black dots on a white background, giving an opaque plate with an array of transparent holes. These were spaced at 0·025 cm., each hole having a diameter of 0·0012 cm., and were arranged on a square with 26 holes along each side. To produce a photograph representing a structure a photographic plate was supported 1 mm. above the pin-holes, and a light, which could be moved about in a horizontal plane 80 cm. below the plate, was switched on for a short time in each of the several positions representing atomic positions in the unit cell. The pin-hole camera gave a reduction in linear dimensions of 800 : 1, and the size of the "unit cell" on which the lamp was moved was, therefore, $0{\cdot}025 \times 800 = 20$ cm.

This method gave good results but suffered from two disadvantages: the pin-hole images are somewhat diffuse, and specks of dust may block the pin-holes and give rise to images of unequal intensities.

It is therefore advantageous to use lenses rather than pin-holes. Small lenses can be made in the following way: if a steel ball-bearing is pressed into a copper

plate, a spherical depression of nearly optical perfection is produced; if, now, a piece of Perspex is pressed at its softening temperature into the copper plate, a spherical lens will be embossed on the surface of the Perspex.

To make a fly's eye we require a regular array of such lenses, and we want, therefore, to make a regular array of depressions in a copper plate. Two ways of obtaining such an array are (1) by packing a large number of balls into a tray and pressing a copper plate on to them, (2) by making each depression individually and moving the copper plate through the repeat distance of the pattern between successive depressions.

The first method suffers from one inherent disadvantage, namely, that it fixes the spacing between the depressions relative to their radius of curvature. The distance between adjacent balls in the tray must be twice their radius, r, and since the lenses will have a focal length of $r/(\mu-1)$, this distance is necessarily $2(\mu-1)$

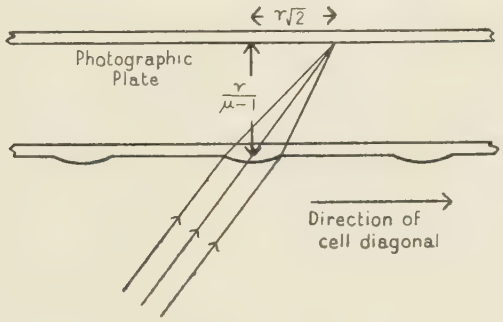


Figure 1. Diagram showing maximum inclination of rays to axes of lenses.

times the focal length of the lenses, where μ is the refractive index of Perspex, equal to 1·49. Therefore rays which would come to a focus at the *corner* of a unit cell (see figure 1) would have to be inclined to the axis at an angle of

$$\tan^{-1}(\mu-1)\sqrt{2}=35°.$$

This is too large an angle between rays and axis for a plano-convex lens of reasonable aperture, and it cannot be reduced except by making $(\mu-1)$ smaller; this could be done by placing another transparent medium over the lenses, but this would introduce complications in construction.

The second method has the disadvantage that the spacing and diameter of the lenses are not so easily kept constant, but with suitable precautions it was found possible to make the variations in these unimportant.

*Construction**

A cylindrical piece of copper 1·5 in. in diameter and 0·5 in. thick was polished on one of its plane faces, first with emery paper and then with metal polish. In order to provide an accurately measurable translatory motion, the copper was screwed firmly, with its polished side upwards, to a lathe tool-rest with two gradu-

* All dimensions are given in inches, since machine tools and ball-bearing balls are calibrated in inches.

ated screw-motions at right angles, which in turn was screwed to the base plate of a drilling machine. A steel ball, 3/16 in. in diameter, was fixed with wax into a depression in the end of a steel rod, which was supported in the drill holder of the machine. A weight was suspended from the arm of the drilling machine and, by lowering this weight slowly, the ball was brought down with a constant thrust (as in a hardness-testing machine) on the copper plate. (The usual rotary motion of the drilling machine was not, of course, used.) The weight was adjusted by trial so that a depression of 0·025 in. diameter was made. The plate was moved 0·05 in. after each depression was made. A slight "burr" was thrown up around the depressions, and it was therefore important not to make them too large in relation to their distance apart.

The depressions were made to lie on a square lattice with a spacing of 0·05 in. and to cover a circular area 1·05 in. in diameter; there were 341 depressions altogether. Their diameters were constant to about 10%, which was sufficiently good for this purpose.

The lenses were then made from this negative by pressing a piece of Perspex between the copper plate and a polished brass plate in a moulding press. The Perspex was originally 0·15 in. thick and was cut to a circle 1·5 in. in diameter. The moulding press was heated to about 140° c., at which temperature the Perspex softened, and was screwed down slightly; about 0·02 in. was sufficient to emboss the lenses. The pressure was not released until the temperature had fallen below 80° c., so that the Perspex was hard again.

The lenses so produced had a numerical aperture of about $f/7{\cdot}5$, and were found by photographic tests to give good definition over the whole field of view employed.

In order that light should pass only through the lenses, it was necessary to make the plane parts of the Perspex opaque. Methods of inking or painting over the plane parts were tried but were found impracticable, as no ink or paint could be found which was opaque in layers as thin as the embossed lenses. Instead, a mask was made by punching holes of the same diameter as the lenses in a piece of black paper. A needle with its end ground flat was used as a punch and the paper was supported on a flat piece of lead. The drilling machine and lathe tool-rest were used, as described above, to place the holes correctly. The mask was stuck to the Perspex with "Durofix", which was applied only round the edges of the fly's eye, to avoid contaminating any of the lenses.

Setting up

In order that a photographic plate may be supported in the focal plane of the lenses, three holes were drilled and tapped near the circumference of the Perspex disc, and three 8-B.A. screws, pointed at their tips, were inserted in these holes and provided with nuts for locking. The fly's eye was fixed by these screws on to a circular brass plate with a hole in its centre large enough to include all the lenses. This was mounted on a horizontal shelf of adjustable height with the lenses downwards and the tips of the screws upwards (figure 2). Below this was a base plate on which a lamp could be moved to various known positions by reference to two scales at right angles. This could be replaced when desired (see below) by an

illuminated ground-glass screen. The whole apparatus must, of course, be used in a darkened room.

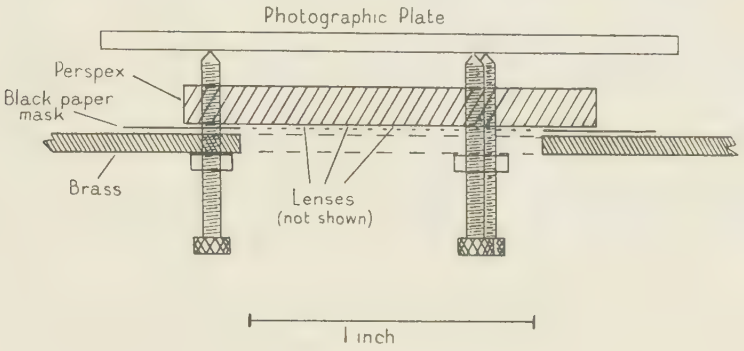


Figure 2. Set-up of fly's eye.

Adjustments

(i) *Focus.* The three screws were adjusted, before the fly's eye was finally set up, by placing the fly's eye on a microscope stage with a piece of ground glass resting on the screws, and turning the screws so that all the images of a distant object were seen in the same plane as the ground glass. A blue filter was used so as to obtain the focus for blue light.

A slight adjustment of the focus was now made to allow for the fact that the photographic plate responded mainly to ultra-violet light. This was done by lowering one of the screws by about half a turn and taking a fly's eye photograph of the lamp in a number of positions. Since the photographic plate is tilted during

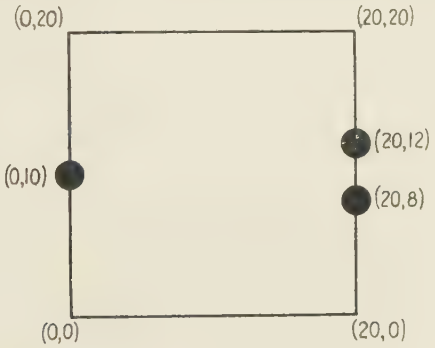


Figure 3. Unit cell and positions of lamp.

this exposure, the images are not in focus all over the plate; by finding those images for which the definition is best, the height of the focal plane could be calculated, and hence the screws were adjusted to the correct height.

(ii) *Scale and orientation.* Two adjustments are now necessary to ensure that the contents of the unit cell shall be reproduced in the fly's-eye photograph at the correct scale and in the correct orientation. This was done as follows: a photograph was taken for which the lamp was switched on in positions whose coordinates were (0,10), (20,8), (20,12), measured in cm., the "unit cell" being 20×20 cm. (figure 3). The fly's eye should, if correctly adjusted, give a pattern as

shown in figure 4 (*a*), where the three black circles represent the spots belonging to one unit cell. A typical out-of-adjustment photograph is represented by figure 4 (*b*),

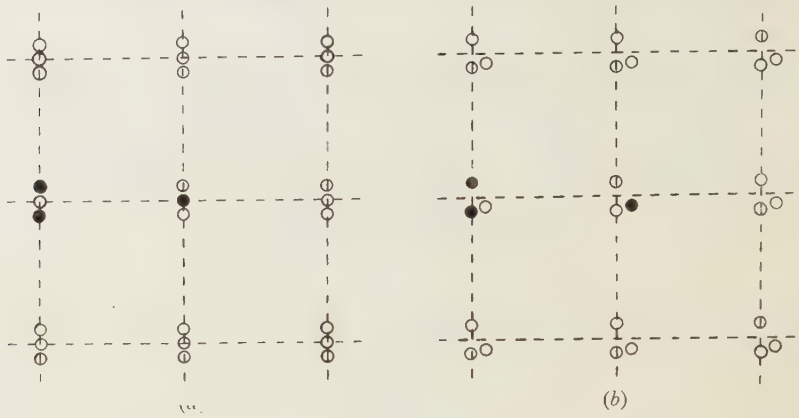


Figure 4. Photographs obtained with (*a*) correct adjustment, (*b*) incorrect adjustment.

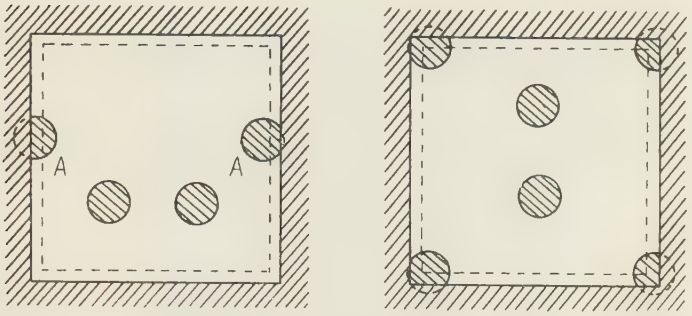
which shows that a rotation of the fly's eye and a reduction of scale are necessary. To reduce the scale, the shelf on which the fly's eye is mounted must be raised.

§ 3. METHOD OF PHOTOGRAPHING A PROPOSED STRUCTURE

The method used with the original pin-hole fly's eye was to switch on the lamp for a suitable length of time in each of the positions at which an atom was to be represented, so that each atom was reproduced as a black spot in each unit cell of the fly's-eye photograph. A negative of this photograph, in which each atom would be represented by a white spot, was then taken, and diffraction spectra from it were observed.

This method may also be used for the lens fly's eye, but a more convenient method is available. If the lamp is replaced by a uniformly illuminated screen, 20 cm. square, and a number of opaque discs are placed on the screen, the fly's-eye photograph will have a black background with light circles on it representing atoms. This method has the advantages that no negative need be taken, and that only one exposure is needed, once the discs have been arranged on the screen, instead of one for each atom.

In practice, a difficulty arises here because the lenses of the fly's eye may not all be accurately spaced, and, therefore, the black squares which are the images of the light square 20×20 cm. may overlap in some places and fail to meet in others. Overlapping does not matter provided the plate is sufficiently over-exposed for all the black part to be practically opaque: the failure of some of the squares to meet may be overcome by making the illuminated squares slightly larger than 20×20 cm., so that all the squares overlap a little. Regions near the edge of the unit cell in the fly's-eye image of the screen are now illuminated twice over, by light from opposite edges of the screen, and no parts are left unilluminated. Any atom that lies on or near the edge of the unit cell must now be represented twice over, as shown at AA in figure 5 (*a*), to ensure that the image of A shall be a complete shadow. Similarly an atom near the corner of the cell must be represented four times over, as shown in figure 5 (*b*).



(*a*) (*b*)

Figure 5. Repetition of atom near edge or corner of unit cell.

§ 4. THE DIFFRACTION APPARATUS

The apparatus for observing the diffraction spectra was similar to that already described by Bragg (1944). It consisted essentially of a mercury-vapour arc lamp with a pin-hole stop, a lens of about a metre focal length to produce a collimated beam, in which the two-dimensional grating was placed, and a similar lens to focus the spectra. Brass tubes were arranged to eliminate extraneous light. The separation of the spectra thus produced was only about 0·3 mm.; an eyepiece or low-power microscope was therefore used for observing the spectra. A green or deep blue filter was found useful for separating a single line of the mercury spectrum; this was almost essential when the spectra were to be photographed.

As the emulsion and the glass of the grating are not of very uniform thickness, the diffraction pattern may be improved considerably by placing the plate between two optical flats and filling the spaces with cedarwood oil.

§ 5. TEST OF FLY'S EYE ON DURENE STRUCTURE

The fly's eye was tried out on the known structure of durene (Robertson, 1933). The unit cell has not a square cross-section, but if all the atomic-position coordinates are referred to equal axes at right angles the intensities of diffraction are not altered. The projection of the structure along the *b*-axis was taken, and since this gives two identical parts to each unit cell, the cell was halved before referring it to equal axes at right angles. Each carbon atom was represented by a disc laid on an illuminated screen as described above, while the hydrogen atoms were neglected. An enlarged reproduction of the fly's-eye photograph is shown in figure 6, while in figures 7 (*a*) and 7 (*b*) are shown the diffraction photographs obtained from this: the first was taken with the fly's-eye photograph enclosed between optical flats and cedarwood oil, while the second was taken without the use of optical flats. The figures under the spots in figure 7 (*a*) are relative intensities calculated from Robertson's data. The agreement is seen to be good when allowance is made for the fact that the discs do not properly represent the carbon atoms, giving a diffracted intensity which does not fall off rapidly enough with angle; also the effect of temperature on intensity is neglected.

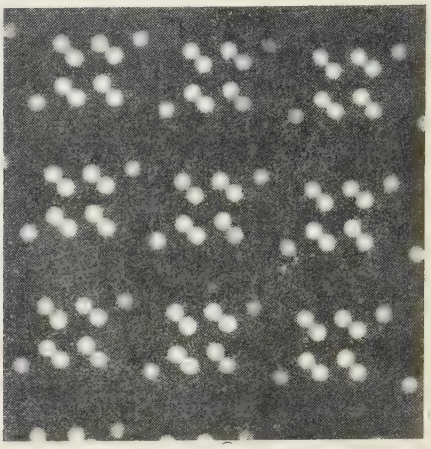
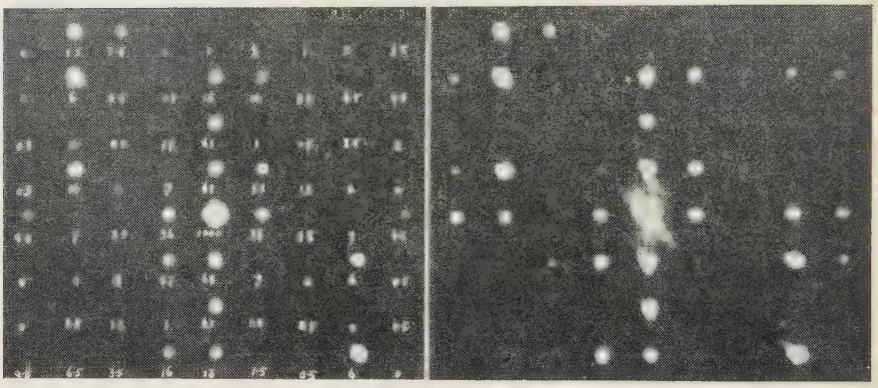


Figure 6. Fly's-eye photograph of durene.



(*a*) (*b*)

Figure 7. Diffraction pattern from durene grating: (*a*) optical flats and cedarwood oil used; (*b*) no optical flats used.

ACKNOWLEDGMENTS

I wish to thank Prof. Sir Lawrence Bragg, F.R.S., under whose guidance the work was started, and Dr. H. Lipson and Dr. G. L. Rogers for many helpful discussions. I am indebted also to the Master and Fellows of Trinity College for financial support through a Senior Rouse Ball Studentship.

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DISCUSSION

Mr. D. E. Thomas. Attempts to follow Dr. Stokes and Prof. Bragg's technique have been made in my laboratory at the Research Department, Woolwich, and one or two minor differences have arisen. The original mould was made with a Vickers hardness-testing machine, the brass block being held in a clamp which could be traversed in two directions at right angles by means of micrometer screws. The technique of making the mask differed considerably. An image of the Perspex fly's eye was made by focusing a

distant small source of light on to a lantern slide. The image was intensified by multiple printing until it gave clear holes on a black background. It was then fitted immediately over the fly's eye.

After forming the diffraction grating it was reduced to one-third (linear dimensions) on a fine resolution plate, and this small grating used to give the diffraction patterns of a point source of monochromatic light. These patterns can be seen with the naked eye and photographed, and are of a convenient size for records.

Dr. R. S. Clay. I suggest that the author could have made the impressions more easily if he had used the group of balls in the frame, and obtained closer grouping of the impressions by moving the frame, after making one impression, an exact fraction of the distance apart of the balls (e.g. a third of the distance apart), then making a second impression, and so on. With nine impressions (in the case assumed) he would get a complete mould.

Prof. A. O. Rankine. Dr. Stokes's paper emphasizes a certain ambiguity in the use of the word " spectra " as between x-ray and ordinary light phenomena. It sounds a little odd to hear of a blue filter being inserted to render the spectra more definite, although the meaning, on contemplation, is evident enough. Must diffraction patterns originating from single wave-length radiation on the one hand, and arrays of images produced by composite wave-length radiation on the other, both be designated by the same term—spectra ?

Author's reply. I am interested in Mr. Thomas's method of making and using his fly's eye. With regard to Dr. Clay's suggestion, I doubt whether the advantage of this method would outweigh the trouble of both fitting balls in a frame and providing the necessary translatory motion ; moreover, a much greater force would be needed in order to produce many indentations at once. It is, incidentally, somewhat difficult to persuade balls to form a square array in a tray ; they always tend to go into a hexagonal pattern.

In answer to Prof. Rankine's point about spectra, my view is that, in x-ray or optical diffraction, one spectrum appears on a photographic plate generally as a series of spots, one for each wave-length of the incident waves ; if these be monochromatic, then a spectrum consists of one spot only. The several spots which arise from the different orders of diffraction may thus be correctly called spectra. It is, however, more precise, though less convenient, to speak of " orders of diffraction " rather than " spectra ".

FLUCTUATION NOISE IN A RECEIVING AERIAL

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Communicated by R. L. Smith-Rose. MS. received 8 *November* 1945

ABSTRACT. The factors which determine the signal/noise ratio at the terminals of a receiving aerial are discussed. The aerial noise considered is the random fluctuation type, consisting of

(i) thermal noise associated with the loss resistance of the aerial, and

(ii) noise associated with the radiation resistance which is induced by the surroundings.

The effective temperature of the radiation resistance is expressed in terms of the temperature distribution of the surroundings and the distribution of power dissipation when the aerial is transmitting. Radiation from the sun and from the Milky Way are briefly discussed, and it is shown that the detection of solar radiation at radio frequencies requires the use of highly directional aerials.

The limitations imposed by the receiver noise on

(*a*) the sensitivity of the reception of signals, and

(*b*) the accuracy of measurement of aerial noise, are discussed and the results presented graphically.

The conclusions are of most practical interest at the higher radio frequencies (above about 20 Mc./s.) where atmospheric noise is negligible.

§ 1. INTRODUCTION

It has been shown (Burgess, 1941) that for an aerial at thermal equilibrium in an enclosure at temperature T, the mean square noise e.m.f. has the value given by Nyquist's theorem,

$$e_n^2 = 4\boldsymbol{k}TRB = 4\boldsymbol{k}T(R_0 + R_r)B, \qquad \ldots\ldots(1)$$

where R is the total resistance and B is the band-width of the receiving system. This noise is composed partly of the intrinsic thermal agitation e.m.f. generated in the loss resistance R_0 of the aerial and partly of the noise received from the surroundings which can be attributed to the radiation resistance R_r.

In practical aerial systems these ideal conditions do not obtain, and the effective temperatures T_0 and T_r of the ohmic and radiation resistances will not in general be equal. The mean square noise e.m.f. is thus

$$e_n^2 = 4\boldsymbol{k}(T_0R_0 + T_rR_r)B. \qquad \ldots\ldots(2)$$

It is assumed throughout that all the noise is of the random fluctuation type, in which the mean square e.m.f. is strictly proportional to band-width.

The present note is supplementary to the earlier paper, and discusses the influence of the aerial characteristics and the receiver noise on the limiting sensitivity of signal reception and on the detection and accuracy of measurement of noise fields.

§ 2. THE EQUIVALENT CIRCUIT OF A RECEIVING AERIAL FOR SIGNAL AND NOISE

The effective height of an aerial for reception is a function of the direction of arrival and polarization of the received wave. If the wave has electric intensity E at some reference point in the aerial system, the induced signal e.m.f. appearing at the terminals is given by

$$e_s = Eh(\theta, \phi, \alpha), \qquad \ldots\ldots(3)$$

where h is a function of the direction of arrival (θ, ϕ) and polarization (α) as well as of frequency and of the choice of terminals.

The effective height for transmission has the same value and the radiation resistance of the aerial can be found readily from Poynting's theorem:

$$R_r = \frac{30}{\lambda^2}\int_0^{2\pi} d\alpha \int h^2 . d\Omega, \qquad \ldots\ldots(4)$$

where λ is the wave-length, while $d\Omega = \sin\theta . d\theta . d\phi$ is the element of solid angle. The equation gives R_r in ohms if λ and h are in metres; M.K.S. units are assumed throughout this paper. It is not customary to take account of the dependence of effective height on polarization, but this is essential in the present analysis

since, in the transmitting condition, an aerial in general produces elliptical polarization at a distant point. Furthermore, the random polarization of noise fields requires integration over all angles of polarization in order to calculate the induced noise e.m.f. in the case of reception.

The directivity or gain of an aerial is a measure of the extent to which its polar diagram is concentrated in one direction and for one polarization. Thus if h_m is the maximum effective height which occurs for this direction and polarization, the directivity can be defined by

$$D = \frac{4\pi^2 h_m^2}{\int_0^{2\pi} d\alpha \int h^2 . d\Omega} . \qquad \ldots\ldots(5)$$

The directivity is unity for a hypothetical omnidirectional aerial giving linear polarization ($h = h_m \sin\alpha$). For linearly polarized aerials the above equation agrees with the usually accepted definition of directivity, e.g. for a Hertzian dipole $D = 1{\cdot}5$ and for a half-wave dipole $D = 1{\cdot}64$.

Combining equations (4) and (5) gives

$$R_r = \frac{120\pi^2 h_m^2}{\lambda^2 D} \text{ (ohms).} \qquad \ldots\ldots(6)$$

The *maximum* available power for such an aerial in a plane wave of electric intensity E volts/metre is

$$P = \frac{E^2 h_m^2}{4R_r} = \frac{E^2 \lambda^2 D}{480\pi^2} \text{ watts}$$
$$= F\lambda^2 D/4\pi \text{ watts,} \qquad \ldots\ldots(7)$$

where $$F = \frac{E^2}{120\pi} \text{ watts/m}^2$$

is the energy flux in the wave.

In practice the resistance R of an aerial is the sum of the radiation resistance R_r and the resistance R_0, which represents the losses in the conductors, the dielectrics and the earth (and in the terminating resistance in the case of a progressive wave system), all at the ambient temperature T_0. It is convenient to write for the ratio of the radiation resistance to the total resistance

$$y = \frac{R_r}{R},$$

where y may for obvious reasons be termed the efficiency. For a resonant dipole y may closely approach unity, but for a short aerial or a progressive wave-system it may be small.

Now the effective noise temperature of a linear two-terminal system whose elements are at different temperatures T_k is readily shown by an application of the Reciprocity Theorem to be given by

$$T' = \frac{\Sigma P_k T_k}{\Sigma P_k} = \frac{\Sigma P_k T_k}{P}, \qquad \ldots\ldots(8)$$

where P_k is the power dissipated in the kth element when a generator is applied to the terminals of the system and P is the total power dissipated.

This equation is directly applicable to an aerial. The loss resistance R_0, which corresponds to the power dissipated in the aerial itself and in its immediate surroundings, is at the ambient temperature T_0. The radiation resistance R_r corresponds to the power which will eventually be dissipated at a relatively great distance from the aerial, and thus its effective temperature cannot be so readily determined. For example, on low radio frequencies (say below 1 Mc./s.), radiation can neither escape from nor enter the space bounded by the earth and the ionosphere, and in this case the effective temperature T_r of the radiation resistance will lie somewhere between the temperature of these boundaries. At higher radio frequencies, where penetration of the ionosphere occurs, the temperature of bodies lying outside it must be taken into account.

The equivalent circuit of the receiving aerial for signal and noise may now be readily constructed. The signal e.m.f. is hE, where h is the effective height for the particular conditions of the incident wave, and will be h_m if adjustment is made for maximum reception. The noise e.m.f.s are $\sqrt{(4\boldsymbol{k}T_0R_0B)}$ in series with the ohmic resistance R_0 and $\sqrt{(4\boldsymbol{k}T_rR_rB)}$ in series with the radiation resistance R_r. The noise e.m.f.s may be combined by ascribing an effective temperature T' to the whole aerial resistance R, where

$$T' = \frac{R_rT_r + R_0T_0}{R} = yT_r + (1-y)T_0. \qquad \ldots\ldots(9)$$

The overall signal/noise ratio at the aerial terminals in the condition of maximum reception is thus

$$\frac{e_s}{e_n} = \frac{h_mE}{\sqrt{4\boldsymbol{k}BR_r\left(T_r + \frac{1-y}{y}T_0\right)}} = \frac{\lambda E}{\sqrt{480\pi^2\boldsymbol{k}BT_r}}\sqrt{\frac{D}{1+\frac{1-y}{y}\frac{T_0}{T_r}}}. \qquad \ldots\ldots(10)$$

The first factor represents the signal/noise ratio of a perfectly efficient aerial ($y=1$) of directivity unity. To render practical interpretation easy, figure 1 shows the field strength for which the signal/noise ratio is unity in a loss-free Hertzian dipole ($D=1{\cdot}5$, $y=1$) with a band-width B of 10 kc./s. and an ambient temperature of 290° K. The corresponding equation is

$$E_1 = \frac{\sqrt{(320\pi^2\boldsymbol{k}BT_r)}}{\lambda} = \frac{0{\cdot}35}{\lambda}\sqrt{\frac{T_0}{T_r}} \qquad \ldots\ldots(11)$$

(μv./m. when the wave-length is in metres).

It is seen from equation (10) that if the noise field is relatively large ($T_r \gg T_0$) the efficiency of the aerial is of secondary importance. It is on account of the prevalence of high noise levels that aerial systems, which theoretically appear to be poor noise collectors of signal energy, are in practice found to be little inferior to systems designed to be of high efficiency.

If the noise radiation is not perfectly diffuse, T_r will itself be a function of the directivity and, in fact, may either increase or decrease as D increases, depending upon whether the polar diagram is concentrated on a region of relatively higher or lower temperature.

§ 3. THE EFFECTIVE TEMPERATURE OF THE RADIATION RESISTANCE

The density of black-body radiant energy at absolute temperature T in a frequency band B is given by the Rayleigh-Jeans law at radio frequencies:

$$U=\frac{8\pi \boldsymbol{k}T}{\boldsymbol{c}\lambda^2}B.$$

The energy flux received from a solid angle $d\Omega$ is given by

$$dF=\frac{dE_n^{\,2}}{120\pi}=\frac{\boldsymbol{c}U}{4\pi}.d\Omega=\frac{2\boldsymbol{k}TB}{\lambda^2}.d\Omega \text{ watt/m}^2,$$

where $dE_n^{\,2}$ is the element of mean square noise field intensity.* Thus the total mean square noise e.m.f. induced in an aerial from its surroundings is given by

$$e_n^{\,2}=\frac{240\pi \boldsymbol{k}B}{\lambda^2}\int_0^{2\pi}\frac{d\alpha}{2\pi}\int Th^2.d\Omega.$$

Therefore the effective temperature of the radiation resistance is given by

$$T_r=\frac{e_n^{\,2}}{4\boldsymbol{k}BR_r}=\frac{\int_0^{2\pi}d\alpha\int Th^2.d\Omega}{\int_0^{2\pi}d\alpha\int h^2.d\Omega} \qquad \ldots\ldots(12)$$

by virtue of equations (5) and (6). This formula for T_r is seen to be formally identical with equation (8) for a linear network, and, in fact, $h^2.d\Omega.d\alpha$ is proportional to the element of power dissipated in a direction $d\Omega$ for angles of polarization between α and $(\alpha+d\alpha)$.

If the source of radiation is a black-body at temperature T which subtends only a small solid angle Ω at the aerial, and furthermore, if the aerial is linearly polarized and directed towards the body, equation (12) assumes the simple form

$$T_r=T\frac{\Omega D}{4\pi}, \qquad \ldots\ldots(13)$$

provided that the aerial directivity D is small compared with $\frac{4\pi}{\Omega}$. To take a specific instance, consider the case of black-body radiation from the sun for which approximately

$$T=6000^\circ\text{K.}, \quad \Omega=7\times10^{-5},$$
$$F=1{\cdot}12\times10^{-13}/\lambda^2 \text{ watts/m}^2 \text{ per Mc./s.},$$
$$T_r=0{\cdot}03D^\circ\text{K.},$$

and thus it will be appreciated that very high directivity of the order of 10^4 is required if the solar radiation is to contribute appreciably to the aerial noise. Furthermore, it is not certain that the sun will radiate as a black-body or that absorption in the intervening space will be negligible, and it is, therefore, possible that the noise field at the aerial will differ from the ideal case calculated above. Measurements of noise at radio frequencies of about 20 Mc./s. by Jansky (1935

* $dE_n^{\,2}$ is *twice* the mean square field measured in any one direction in the wave-front, the factor of 2 taking into account the two independent orthogonal polarizations which contribute equally to the energy flux of a randomly polarized wave.

and 1937) did not reveal any evidence of solar radiation, nor did those of Reber (1940 and 1942) at higher frequencies (160, 900 and 3300 Mc./s.).*

Jansky (1935 and 1937), Reber (1940 and 1942), and Franz (1942) have established that an intense source of fluctuation noise exists in a fixed region in space in the direction of the Milky Way. The exact nature and origin of this radiation has not yet been conclusively established, but if it is thermal, the temperatures at the source must be very high (probably greater than 100,000° K.). The discussion by Henyey and Keenan (1940) is of interest in this connection.

It is suggested that refined measurements of noise radiation at relatively high frequencies for which the ionosphere and the earth's atmosphere are sensibly transparent, and at which directive aerials are readily constructed, would provide much useful information regarding the temperature of, and processes in, regions of outer space

§ 4. THE SIGNIFICANCE OF RECEIVER NOISE

If an aerial is attached to an ideal receiver which produces no noise, the output signal/noise ratio (measured at a point prior to detection) would be the same as that at the aerial. Similarly any source of noise radiation whose field acts on the aerial could be detected provided that sufficient amplification were available.

In practice a receiver contributes to the output noise level, so resulting in a reduction of signal/noise ratio in the case of signal reception or in masking of the aerial noise where measurement of the latter is desired.

A convenient measure of receiver noise is the "noise factor" usually denoted by the symbol N. A receiver of noise factor N connected to a source of signal, whose resistive component R of impedance is at temperature T_0, will give a signal/noise ratio which is $1/N$ times that given by an ideal noise-free receiver of the same band-width connected to the same source. The noise e.m.f. of the receiver itself referred to the aerial terminals is, therefore,

$$e_r^2 = 4\boldsymbol{k}T_0(N-1)RB. \qquad \ldots\ldots(14)$$

(In practice it is more convenient to express the noise factor as $10\log_{10}N$ decibels, but in this equation and those that follow the numerical power ratio is used.) It is seen from this equation that the noise of the receiver can be expressed as an equivalent field intensity by taking $(N-1)$ for T_r/T_0 in figure 1.

The loss in signal/noise ratio when the receiver is joined to an aerial at effective temperature T' will not in general be N, but will be given by

$$N' = 1 + \frac{e_r^2}{4\boldsymbol{k}T'RB} = 1 + (N-1)\frac{T_0}{T'}, \qquad \ldots\ldots(15)$$

which is greater or less than N according as T' is less or greater than T_0. In figure 2, the effective noise factor N' is shown as a function of T'/T_0 for various values of N. It is readily seen that the values of T'/T_0 which occur in practice result in a considerable improvement in the effective noise factor, e.g. if for a given receiver $N = 25$ (14 db.), then for $T'/T_0 = 10$ it is found that $N' = 5{\cdot}3$ db.,

* Since the present paper was written, Reber (1944) and Southworth (1945) report measurements of solar noise, using highly directional aerials which give intensities of the same order as the black-body value calculated above.

while for $T'/T_0 = 100$, $N' = 0{\cdot}9$ db. This serves to explain why in practice an improved design of receiver or aerial frequently fails to give the expected improvement in sensitivity.

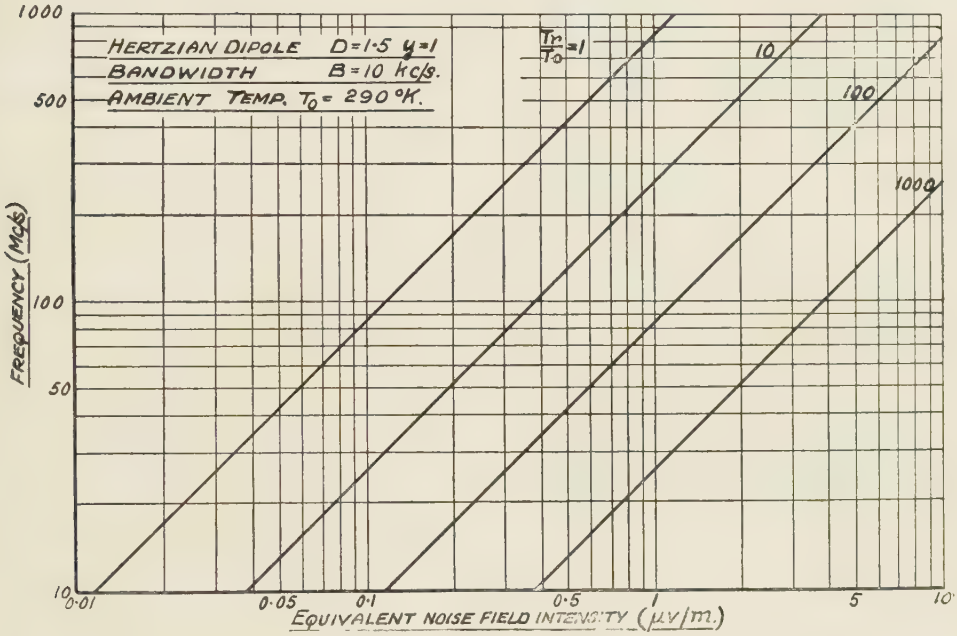

Figure 1. Equivalent noise field intensity for a dipole receiving radiation of temperature T_r.

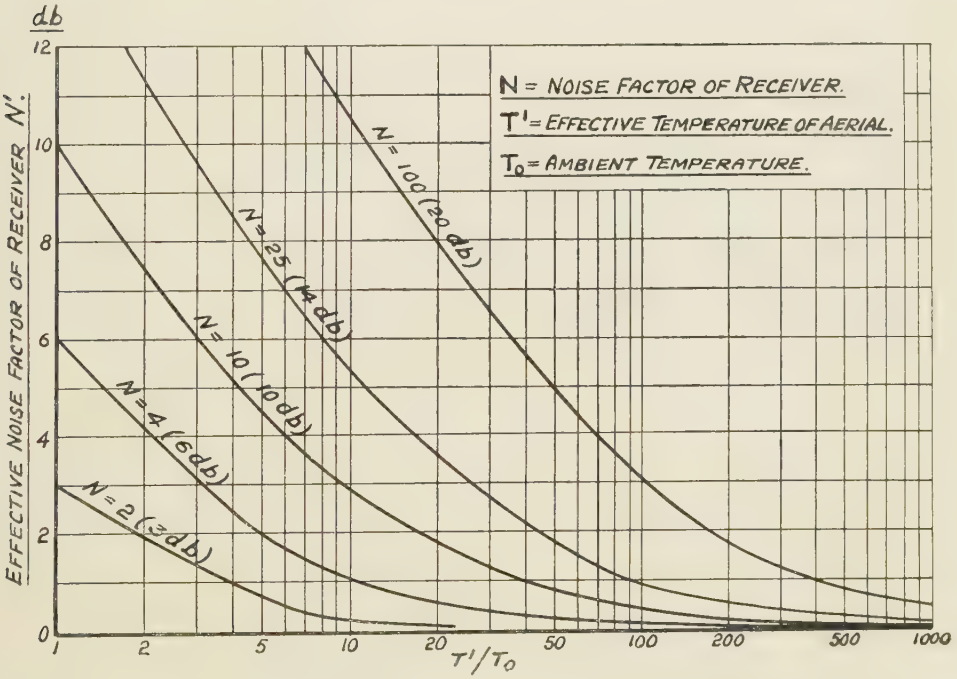

Figure 2. The influence of the aerial noise on the effective noise factor of a receiver.

The effect of receiver noise on the observation of aerial noise can readily be estimated. A convenient method of determining the aerial noise (i.e. T') is to

measure the ratio n of the output noise powers when the receiver is connected in turn to the aerial (at T') and to the dummy (at T_0):

$$n = \frac{4kT'RB + e_r^2}{4kT_0RB + e_r^2} = 1 + \frac{T' - T_0}{NT_0}. \qquad \ldots\ldots(16)$$

Figure 3 shows the noise ratio n as a function of T'/T_0 for various values of N. The accuracy of the measurement of T' can be expressed by means of the differential ratio:

$$\frac{\Delta T'}{T'} \Big/ \frac{\Delta n}{n} = 1 + (N-1)\frac{T_0}{T'} = N'. \qquad \ldots\ldots(17)$$

Thus the fractional uncertainity of T' is always greater than that in n, and if $\Delta T'/T'$ becomes the order of unity it may be said that the aerial noise is not measurable.

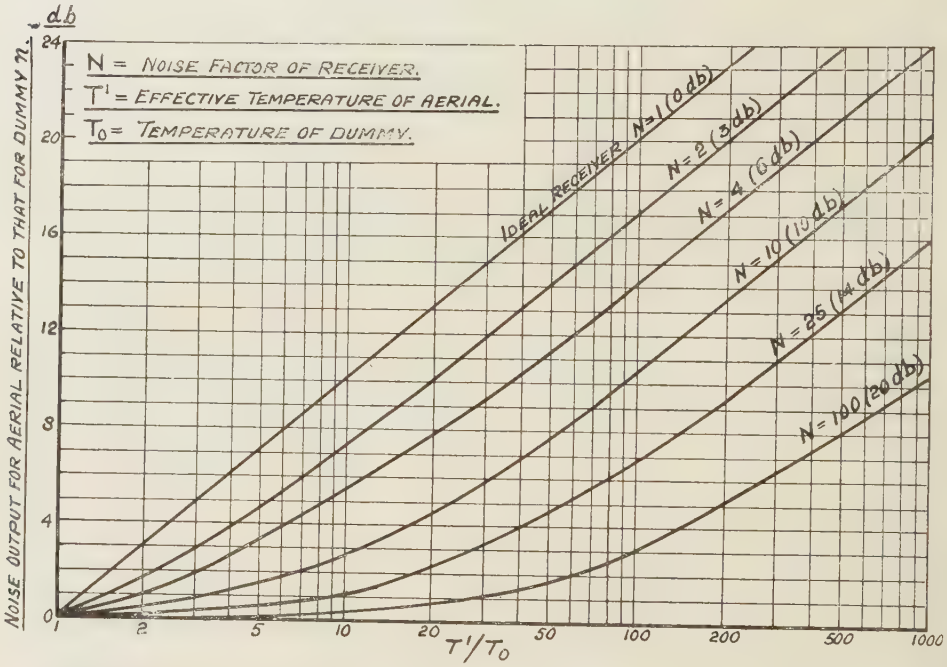

Figure 3. The influence of receiver noise on the measurement of aerial noise.

As the frequency of measurement increases, it is possible to obtain higher aerial directivity for a given aperture and thus the effective aerial temperature T' due to noise from a single source (such as the sun) is raised. This improvement is, however, slightly offset by the increase in the receiver noise factor at the higher frequencies.

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MEASUREMENT OF DIFFUSION AND THERMAL CONDUCTIVITY "CONSTANTS" IN NON-HOMOGENEOUS MEDIA, AND IN MEDIA WHERE THESE "CONSTANTS" DEPEND RESPECTIVELY ON CONCENTRATION OR TEMPERATURE

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MS. received 1 *October* 1945

ABSTRACT. Equations have been derived for the steady-state distributions of temperature or concentration across membranes in the form of plates, tubes or spherical shells, when the thermal conductivity, or diffusion, "constants" are functions respectively of temperature, or solute concentration, in the diffusion medium, and also where these "constants" are functions of the positional co-ordinates. Equations are similarly given for the quantity of heat or solute transported through the membranes.

Procedures based on these equations are given for measuring diffusion and thermal conductivity "constants" and their dependence upon concentration and temperature respectively, or on positional co-ordinates. All observational data are based only on the steady state of flow, thus avoiding the need to deal with equations such as $\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right)$, which have so far not been integrated. The methods are suitable for measurements of flow of liquids, vapours or dyestuffs through natural and artificial plastics, of gases and liquids through zeolitic media, in surface migration of adatoms on metals, and similar problems.

§ 1. INTRODUCTION

The focus of attention in quantitative studies of the flow of matter or of heat has long been Fick's law :

$$\frac{\partial c}{\partial t}=D\nabla^2 c, \qquad \ldots\ldots(1)$$

$$\frac{\partial T}{\partial t}=D\nabla^2 T, \qquad \ldots\ldots(1a)$$

where c and T are concentration and temperature respectively, and D is the diffusion or thermal diffusivity constant. It is now clear that D is often not a

constant at all, so that this law is a limiting law only, just as in other spheres are the Langmuir isotherm, the Debye-Hückel equation or Raoult's law. Equation (1) must then be replaced by the following equation, which allows for the non-constancy of D:

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D\,\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\,\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\,\frac{\partial c}{\partial z}\right). \qquad \ldots\ldots(2)$$

No exact solution of equation (2) has been found for any boundary conditions, although in several instances approximate solutions have been derived (e.g. Hopkins, 1938). Variants of a graphical method have been evolved (Matano, 1933; Eversole, Peterson and Kindsvater, 1938) for determining D as a function of c from the equation

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D\,\frac{\partial c}{\partial x}\right). \qquad \ldots\ldots(2a)$$

The method has been used by Wells and Mehl (1941), Matano (1933), Mehl (1937) and Rhines and Mehl (1937) for diffusion in metallic media and for diffusion of salts in solutions by Eversole *et al.*, 1941, 1942).

In general D may be a function of concentration of the diffusing species, or a function of the positional co-ordinates, x, y and z. D depends on c when metals and non-metals diffuse into metals (see above references; also Wells and Mehl, 1940; Smoluchowski, 1942, 1943; Barrer, 1941 a); salts diffuse in solution (Eversole *et al.*, 1941, 1942); gases and liquids diffuse in zeolitic media (Barrer, 1941 b; Barrer and Ibbitson, 1944); adatoms diffuse on metallic surfaces (Langmuir, 1932, 1934; Langmuir and Kingdon, 1925; Bosworth, 1935, 1937); and liquids or vapours are imbibed by polymers which thereby swell (Daynes, 1937; Barrer, 1941 c; Garvie and Neal, 1938; King, 1945). D depends on positional co-ordinates when heat or matter flows in physically or chemically inhomogeneous media. The unequal thermal gradients which arise at different places on the earth's crust (Bullard, 1945), or the permeability of inhomogeneous rock strata to fluids such as petroleum, afford examples of interest. Diffusion of thorium along a tungsten filament where a temperature gradient exists along the filament is a more complex example, for here D depends both on the thorium concentration (Langmuir, 1932, 1934) and also on the positional co-ordinate, x. Dependence on x in such problems arises independently of dependence on c because D is a function of $T\left(D = D_0 e^{-\frac{E}{RT}}\right)$ and T depends on x.

The rather laborious character of the graphical method of measuring D as a function of c, and the absence of a procedure when $D = f(x, y, z)$, render it important to evolve new simple procedures. The theory of diffusion in condensed phases often awaits methods which permit data to be readily obtained. In this paper procedures are developed for measuring D when this "constant" depends either on concentration or on the positional co-ordinates. The new methods require observation of the stationary state of flow only, with a corresponding increase in directness and practicality. They apply equally to the diffusion of heat or of matter.

§2. THE FUNDAMENTAL EQUATIONS

In Cartesian co-ordinates, equation (2) is the fundamental expression for material diffusion; its counterpart for flow of heat is

$$C_p\rho \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right). \qquad \ldots\ldots(2b)$$

In these equations D has the dimensions l^2t^{-1}, and K, the *thermal conductivity constant*, the dimensions $m^1l^{-1}t^{-1}$. Also C_p is the heat capacity at the point (x, y, z), and ρ the density at this point. Equation (2) is thus a particular case of (2 *b*), where $C_p\rho$ is independent of the dependent or independent variables. However, when D and K are true constants, equations (2) and (2 *b*) reduce to equations (1) and (1 *a*) respectively, the *thermal diffusivity constant* $D = K/C_p\rho$ in equation (1 *a*) now having the same dimensions as the *diffusion constant* D in equation (1).

Transformation of equations (2) and (2 *b*) to cylindrical co-ordinates gave for flow of heat and matter, respectively,

$$C_p\rho \cdot \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(K\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right), \qquad \ldots\ldots(3)$$

$$\frac{\partial c}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial c}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial c}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial c}{\partial z}\right), \qquad \ldots\ldots(3a)$$

Transformation to polar co-ordinates gave similarly

$$C_p\rho \cdot \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2K\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta K\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(K\frac{\partial T}{\partial \phi}\right), \qquad \ldots\ldots(4)$$

$$\frac{\partial c}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2D\frac{\partial c}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta D\frac{\partial c}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial c}{\partial \phi}\right). \qquad \ldots\ldots(4a)$$

Many important problems concern diffusion in cylinders and spheres with concentration or temperature independent of θ and z (for cylinders) or of θ and ϕ (for spheres), so that the corresponding equations follow from equations (3) and (4) on omitting terms in these variables. In the steady state in which we are interested $\frac{dc}{dt} = \frac{dT}{dt} = 0$, and for heat flow, for example, one has

$$0 = \frac{d}{dr}\left(r^2K\frac{dT}{dr}\right) \quad \text{(sphere)}, \qquad \ldots\ldots(5)$$

$$0 = \frac{d}{dr}\left(rK\frac{dT}{dr}\right) \quad \text{(cylinder)}. \qquad \ldots\ldots(6)$$

To these may be added the equation for heat flow in the x-direction only:

$$0 = \frac{d}{dr}\left(K\frac{dT}{dr}\right) \quad \text{(plate)}, \qquad \ldots\ldots(7)$$

The corresponding equations for diffusion of matter follow at once by substituting D for K and c for T.

The equations (5), (6) and (7) may be integrated readily. Thus for equation (5)

$$r^2D\frac{dc}{dr}, \quad \text{or} \quad r^2K\frac{dT}{dr}=A, \qquad \ldots\ldots(8)$$

where A is a constant. Now D (or K) may be a function of c or of $r(x)$. In the former case one has quite generally

$$D=D_0[1+f(c)] \qquad \ldots\ldots(9)$$

and in the latter

$$D=D_0[1+f(r)]. \qquad \ldots\ldots(10)$$

In equation (9), $f(c)$ is conventionally chosen so that D_0 is the value of D when $c\to 0$; in equation (10), similarly, D_0 is the value of D at a chosen face $r=r_1$. In either case the variables may be separated, and one obtains for the steady-state distribution of *solute* across a spherical shell:

$$D_0\int[1+f(c)]dc=-\frac{A}{r}+B \qquad \ldots\ldots(11)$$

when D is a function of c, and

$$D_0c=\int\frac{dr}{r^2[1+f(r)]}+B \qquad \ldots\ldots(12)$$

when D is a function of r. Similarly for the distribution across a hollow cylindrical tube

$$D_0\int(1+f(c))dc=A\ln r+B, \qquad \ldots\ldots(13)$$

$$D_0c=A\int\frac{dr}{r[1+f(r)]}+B, \qquad \ldots\ldots(14)$$

while for a plate

$$D_0\int[1+f(c)]dc=Ax+B \qquad \ldots\ldots(15)$$

$$D_0c=A\int\frac{dx}{[1+f(x)]}+B. \qquad \ldots\ldots(16)$$

§ 3. DETERMINATION OF D_0 AND $f(c)$ WHEN $D=D_0[1+f(c)]$

Equations (11), (13) and (15) all provide means of measuring both D_0 and $f(c)$ when $D=D_0[1+f(c)]$.

When the boundary conditions for diffusion across the plate are

$$c=c_1 \text{ for all } t \text{ at } x=0,$$
$$c=c_2 \text{ for all } t \text{ at } x=1,$$

equation (15) for the stationary state becomes

$$\frac{c_1+F(c_1)-(c+F(c))}{c_1+F(c_1)-(c_2+F(c_2))}=\frac{x}{l}, \qquad \ldots\ldots(17)$$

where* $F(c)=\int f(c)\,dc$. Similarly for the tube or spherical shell with boundary conditions

$$c=c_1 \text{ for all } t \text{ at } r=r_1,$$
$$c=c_2 \text{ for all } t \text{ at } r=r_2,$$

* Integration is between the limits 0 and c. $F(c)$ thus tends to zero as $c\to 0$, the laws of flow in the limit then reducing to the simple Fick laws in accord with experiment.

one obtains from equation (13) (the tube)

$$\frac{c_1+F(c_1)-(c+F(c))}{c_1+F(c_1)-(c_2+F(c_2))}=\frac{\ln r_1-\ln r}{\ln r_1-\ln r_2} \qquad \ldots\ldots(18)$$

and from equation (11) (the spherical shell)

$$\frac{c_1+F(c_1)-(c+F(c))}{c_1+F(c_1)-(c_2+F(c_2))}=\frac{r_2}{r_1-r_2}\cdot\frac{r_1-r}{r}. \qquad \ldots\ldots(19)$$

Thus the steady state concentration distributions depend on the way in which D depends on concentration. From this distribution $F(c)$ and so $f(c)=\frac{d}{dc}(F(c))$ may be derived. Alternatively, one may derive $\frac{dc}{dx}$, which, when $c_2=0$, is (from equation (17) for example) $-\frac{dc}{dx}=\frac{c_1+F(c_1)}{l(1+f(c))}$. Then on plotting $-\frac{dc}{dx}$ as a function of c, one sees the intercept on the axis of x to be $\frac{c_1+F(c_1)}{l}$, and from values of $\frac{dc}{dx}$ between c_1 and 0, $f(c)$ easily follows.

D_0 in equation (9) can now be obtained by measuring the steady-state flow of solute through the medium. This flow per unit area and per unit time is

$$P=-\left(D\frac{dc}{dx}\right)_{x=l}. \qquad \ldots\ldots(20)$$

By differentiation of equation (17) with respect to x one finds $\frac{dc}{dx}$, and then by integration of $\left(D\frac{dc}{dx}\right)_{x=l}$ from 0 to t the quantity Q of solute passing in time t through a membrane of area A is found:

$$Q=A\int_0^t P\,dt=\frac{D_0}{l}\left[c_1+F(c_1)-(c_2+F(c_2))\right]At. \qquad \ldots\ldots(21)$$

For a tube of length L a similar procedure gives

$$Q=2\pi r_2 L\int_0^t P\,dt=2\pi LD_0\left[\frac{c_1+F(c_1)-(c_2+F(c_2))}{\ln r_1-\ln r_2}\right]t, \qquad \ldots\ldots(22)$$

and for a spherical shell

$$Q=4\pi r_2^2\int_0^t P\,dt=4\pi\frac{r_1r_2}{r_1-r_2}D_0[c_1+F(c_1)-(c_2+F(c_2))]\,t. \qquad \ldots\ldots(23)$$

Since $F(c)$ has previously been determined using equations (17), (18), or (19), D_0 can be evaluated in the appropriate equation (21), (22), or (23).

An alternative procedure for determining D_0 and $f(c)$ makes use only of equations (21), (22) or (23). When $c_2 \ll c_1$, equation (21), for instance, becomes

$$\frac{Ql}{tA}=D_0[c_1+F(c_1)], \qquad \ldots\ldots(21a)$$

where all quantities on the left-hand side of equation (24 *a*) are measured. If, now, c_1 is made small, departures from the simple Fick laws tend to zero, so that

$$\lim_{c_1 \to 0} \frac{Ql}{tAc_1} = D_0. \qquad \ldots\ldots (21\,b)$$

Next $F(c)$ is given by the relation

$$\frac{Ql}{tAD_0} - c_1 = F(c_1), \qquad \ldots\ldots (24)$$

the left-hand side of which is determined for a range of values of c. Finally

$$f(c) = \frac{d}{dc}(F(c)).$$

§ 4. DETERMINATION OF D_0 AND $f(x)$ WHEN $D = D_0(1+f(x))$

We now return to consideration of equations (12), (14) and (16) for the steady-state distribution of c when $D = D_0[1 + f(x)]$. If the integral on the right-hand side of each of these equations be denoted by I, and by I_1 and I_2 at each boundary surface, the constants A and B may be eliminated to give for the boundary conditions in the previous section

$$\frac{c_1 - c}{c_1 - c_2} = \frac{I_1 - I}{I_1 - I_2}. \qquad \ldots\ldots (25)$$

Moreover, the quantity Q of solute diffusing through the outgoing surface of a plate of area A is

$$Q = A\int_0^t -\left(D\frac{dc}{dx}\right)_{x=l} dt = -AD_0 t \cdot \frac{c_1 - c_2}{I_1 - I_2}, \qquad \ldots\ldots (26)$$

while for a hollow tube of length L

$$Q = 2\pi r_2 L\int_0^t -\left(D\frac{dc}{dr}\right)_{r=r_2} dt = -2\pi L \cdot \frac{c_1 - c_2}{I_1 - I_2} \qquad \ldots\ldots (27)$$

and for the spherical shell

$$Q = 4\pi r_2^2\int_0^t -\left(D\frac{dc}{dr}\right)_{r=r_2} dt = -4\pi D_0 t \cdot \frac{c_1 - c_2}{I_1 - I_2}. \qquad \ldots\ldots (28)$$

Then to determine D_0 and $f(x)$ for the plate, for example, the steady-state concentration distribution can be measured, and from it the value of

$$\frac{dc}{dx} = \frac{c_1 - c_2}{I_1 - I_2}\frac{1}{(1+f(x))}.$$

On plotting $\frac{dc}{dx}$ against x, its value at $x = 0$ is $\frac{c_1 - c_2}{I_1 - I_2}$, and from its values for $0 < x < 1$, $f(x)$ may be found. Equation (26) then serves to determine D_0.

Details given in this and the previous section apply equally well to the determination of K_0 and $f(T)$ or $f(x)$ in the analogous problem of heat flow.

§ 5. CHARACTER OF STEADY-STATE CONCENTRATION DISTRIBUTION WHEN D DEPENDS ON c OR ON x

In equations (9) or (10) the functions $f(c)$ and $f(x)$ (or $f(r)$) must be such that D is never negative or infinite. For a suitable choice of constants appropriate functions include

$$f(c) = \pm ac^n \pm bc^m \pm dc^p \ldots . \text{(exponents positive or zero)}, \quad \ldots\ldots (29)$$

$$f(c) = \pm \frac{ac}{1+bc} \quad \ldots\ldots (30)$$

$$f(c) = \pm ae^{\pm bc}, \quad \ldots\ldots (31)$$

$$f(c) = \pm ac^n e^{\pm b/c} \quad (n \geqslant 0). \quad \ldots\ldots (32)$$

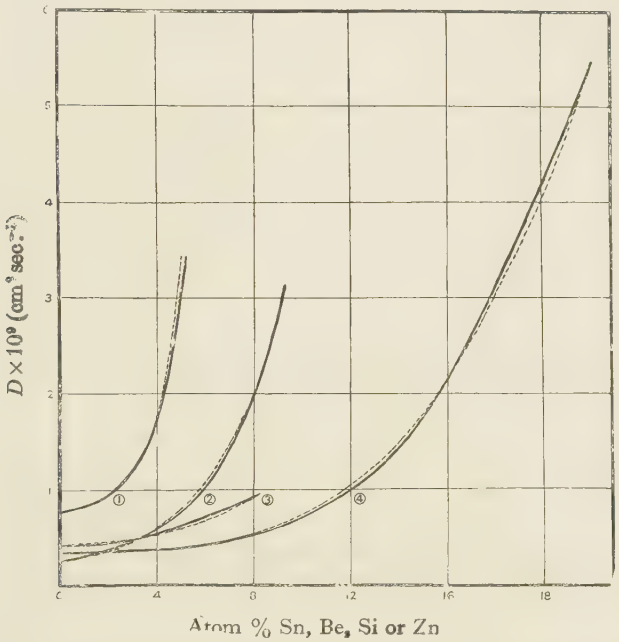

Figure 1 *a*. Experimental (Rhines and Mehl, 1938) (full curve) and calculated (dotted curve) relations between D and c. Calculated curves employ the exponential functions $D=D_0(1+ae^{bc})$ and $D=D_0(1+ac^3e^{-b/c})$.

Calculated Curve (1): Sn—Cu. $D=7{\cdot}7 \times 10^{-10}(1+4{\cdot}46\times10^{-2}\,e^{0{\cdot}862c})$ cm.^{2} sec.$^{-1}$.
,, Curve (2): Si—Cu. $D=2{\cdot}31\times10^{-10}(1+0{\cdot}306\,e^{0{\cdot}402c})$ cm.^{2} sec.$^{-1}$.
,, Curve (3): Be—Cu. $D=4{\cdot}23\times10^{-10}(1+7{\cdot}08\times10^{-2}\,e^{0{\cdot}352c})$ cm.^{2} sec.$^{-1}$.
,, Curve (4): Zn—Cu. $D=3{\cdot}46\times10^{-10}\left(1+1{\cdot}51\times10^{-3}c^3\,e^{-\frac{2{\cdot}304}{c}}\right)$ cm.^{2} sec.$^{-1}$.

Figures 1 *a* and 1 *b* give actual and calculated D-c curves according to several of these equations, and illustrate the degree of success obtained with them.

The steady-state concentration distribution* across a plate of the diffusion medium is given by equation (20), which for $c_2 \sim 0$ becomes

$$c + F(c) = \left(1 - \frac{x}{l}\right)[c_1 + F(c_1)]. \quad \ldots\ldots (17\,a)$$

* In the absence of rate-controlling phase-boundary processes (For a discussion of these processes see Barrer, 1941 a).

It is convenient to choose units for c such that $c_1 = 1$; then in equations (29) to (32), $0 < c < 1$. Also, if the solute has a limited solubility in the diffusion medium, c in these equations may conveniently be replaced by c/c_s where c_s denotes the saturation concentration. Using these conventions, figures 2 *a* and 2 *b* give typical concentration distributions for some of the functions of equations (29) to (32). When D increases as c increases, equilibrium distributions are convex away from the axis of x; but when D decreases with increasing c, the distributions are concave towards that axis. When D at first increases and then decreases, or vice versa,

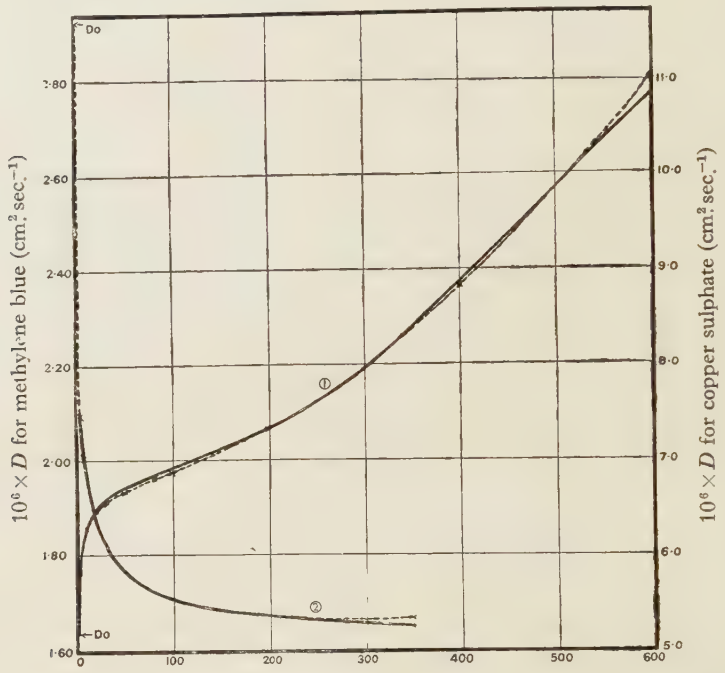

Figure 1 *b*. Experimental (Eversole *et al.*, 1941, 1942), (full curve) and calculated (dotted curve) relations between D and c. Calculated curves employ the functions $D = D_0(1 + ac^n + bc^m + \ldots)$.

Calculated curve 1: Methylene blue—H_2O.
$$D = 1{\cdot}635 \times 10^{-6}(1 + 0{\cdot}0824c^{1/4} - 0{\cdot}00128c + 0{\cdot}0000738c^{3/2}) \quad \text{cm}^2\,\text{sec.}^{-1}.$$

Calculated curve 2: Copper sulphate—H_2O.
$$D = 11{\cdot}65 \times 10^{-6}(1 - 0{\cdot}00311c^{1/4} + 0{\cdot}002645c^{1/3}) \quad \text{cm}^2\,\text{sec.}^{-1}.$$

with increasing c, the behaviour is less predictable (e.g. figure 2 *b*, curve 3). Curves 4 and 6 in figure 2 *a* give the extreme range of steady-state distributions of concentration for a pure zeolitic diffusion process where (*cf.* equation (29)) $D = D_0(1 - c/c_s)$.

Equilibrium distributions when $D = D_0[1 + f(x)]$ follow from equation (25). Two typical values for $f(x)$ will be considered:

$$f(x) = ax, \qquad \ldots\ldots(33)$$

$$f(x) = bx + ax^2. \qquad \ldots\ldots(34)$$

When $f(x)=ax$, I in equation (25) has the values

$$I=\frac{1}{a}\ln(1+ax) \quad \text{(for plate)}, \qquad \ldots\ldots(35)$$

$$I=-\ln\frac{1+ar}{r} \quad \text{(for tube)}, \qquad \ldots\ldots(36)$$

$$I=-\frac{(1+ar)}{r}+a\ln\frac{(1+ar)}{r} \quad \text{(for spherical shell)}, \qquad \ldots\ldots(37)$$

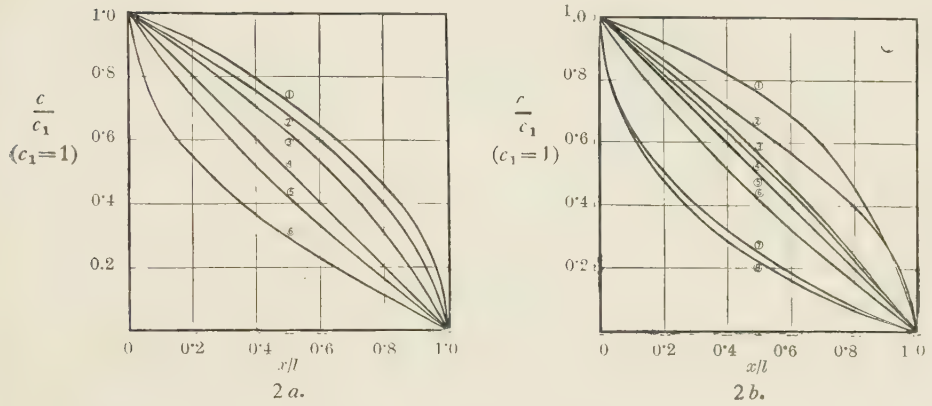

Figure 2 *a*. Typical steady-state concentration distributions across a membrane when $D=D_0(1+f(c))$.

Curve (1): $f(c)=ac$; $a=100$.
Curve (2): $f(c)=ac$; $a=10$.
Curve (3): $f(c)=ac$; $a=2$.
Curve (4): $f(c)=0$ (simple Fick law obeyed).
Curve (5): $f(c)=-ac$; $a=0{\cdot}5$.
Curve (6): $f(c)=-ac$; $a=1{\cdot}0$.

Figure 2 *b*. Typical steady-state concentration distributions across a membrane when $D=D_0(1+f(c))$.

Curve (1): $f(c)=ae^{bc}$; $a=1$, $b=3$.
Curve (2): $f(c)=\dfrac{ac}{1+bc}$; $a=100$, $b=1$.
Curve (3): $f(c)=-ac^{1/2}+bc^2$; $a=1$, $b=2$.
Curve (4): $f(c)=\dfrac{ac}{1+bc}$; $a=1$, $b=1$.
Curve (5): $f(c)=0$ (simple Fick law obeyed).
Curve (6): $f(c)=-\dfrac{ac}{1+bc}$; $a=0{\cdot}9$, $b=1$.
Curve (7): $f(c)=-ac^{1/2}$; $a=1$.
Curve (8): $f(c)=-ac^{1/4}$; $a=1$.

and values for I_1 and I_2 follow on setting $x=0$ and $x=1$ for the plate, or $r=r_1$ and r_2 for the tube and spherical shell. When $f(x)=bx+ax^2$, several values of

$$I=\int\frac{dx}{1+bx+ax^2}$$

result for different values of a and b:

$$\left.\begin{aligned} I &= \frac{2}{\sqrt{4a-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4a-b^2}} \quad (4a>b^2), \\ I &= \frac{1}{\sqrt{b^2-4a}} \ln \frac{2ax+b-\sqrt{b^2-4a}}{2ax+b+\sqrt{b^2-4a}} \quad (b^2>4a), \\ I &= -\frac{2}{2ax+b} \quad (b^2=4a), \end{aligned}\right\} \text{(for plate)} \ldots\ldots (38)$$

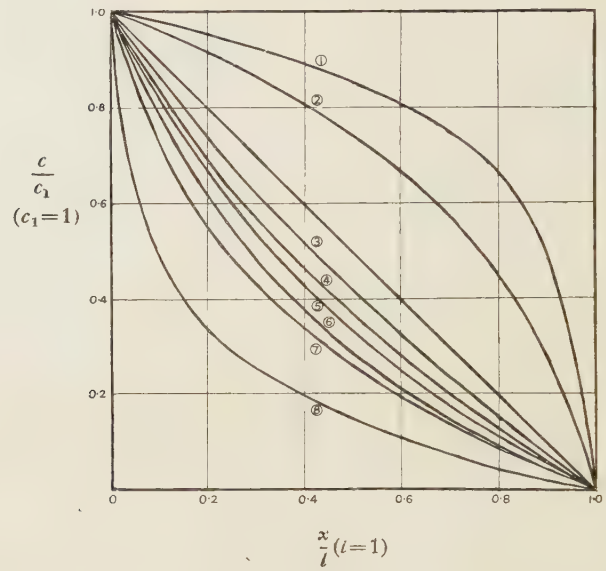

Figure 3. Typical steady-state concentration distributions when $D=D_0(1+f(x))$. The straight line corresponds to thes simple Fick law ($f(x)=0$).

Curve (1): $f(x)=-ax$; $a=0{\cdot}99$.
Curve (2): $f(x)=-ax$; $a=0{\cdot}90$.
Curve (3): $f(x)=ax$; $a=1{\cdot}0$.
Curve (4): $f(x)=ax$; $a=2{\cdot}0$.
Curve (5): $f(x)=bx+ax^2$; $a=1$, $b=2$.
Curve (6): $f(x)=bx+ax^2$; $a=2{\cdot}25$, $b=3$.
Curve (7): $f(x)=ax$; $a=9$.
Curve (8): $f(x)=ax$; $a=99$.

and I_1 and I_2 follow as before. Similarly, when $f(r)=br+ar^2$, the value of

$$I=\int \frac{dr}{r(1+br+ar^2)}$$

for the tube is

$$I=\tfrac{1}{2}\ln\left(\frac{r^2}{1+br+ar^2}\right)-\frac{b}{2}I_{\text{plate}}, \qquad \ldots\ldots (39)$$

where I_{plate} denotes the appropriate integral for the plate but with r replacing x.

Finally, for the spherical shell,

$$I=\int \frac{dr}{r^2(1+br+ar^2)}$$

is $$I = \frac{b}{2} \ln \frac{1+br+ar^2}{r^2} - \frac{1}{r} + \frac{b^2-2a}{2} I_{\text{plate}}. \qquad \ldots\ldots(40)$$

Figure 3 shows some steady state concentration distributions across a plate of unit thickness when $c_1 = 1$ and $c_2 = 0$; and $D = D_0(1+ax)$ (curves 8, and 1 to 5) and $D = D_0(1+bx+ax^2)$ (curves 6 and 7). When D is an increasing function of x the curves are concave towards the axis of x, and when D is a decreasing function of x the $c : x$ curves are convex away from this axis.*

There is little material available for quantitative interpretation according to methods developed in this paper. In rubber, steady-state concentration distributions of water concave towards the x-axis (Taylor, Herman and Kemp, 1936), suggest that D decreases as the moisture content grows.† There seems little regularity in available data, however, for in other polymers, such as viscose silk and wool, D for certain diffusing species increases with c (Garvie and Neal, 1938; King, 1945).

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* Also for equal values of c_2, c_1, and l, whenever the equilibrium concentration distribution is convex away from the x-axis, the membrane is more permeable than predicted by Fick's law with $D = D_0$; and when this distribution is concave towards the x-axis the membrane is less permeable than predicted by Fick's law with $D = D_0$.

† In this instance an alternative explanation is possible. See e.g. Barrer (1941c).

OBITUARY NOTICES

JOHN EDMUND CALTHROP

By the sudden death in June 1945 of J. E. Calthrop, Senior Lecturer in Physics, Queen Mary College has lost one of the most loyal members of its staff, and his fellow-teachers have lost a singularly gentle and unexacting friend.

Calthrop was born in Lincoln in July 1891. He took his London B.Sc. degree in 1912 as a student at the Borough Road Training College, and after two years spent in teaching science at the Lincoln Technical School he enlisted in the Honourable Artillery Company. He was severely wounded at Beaumont Hamel in November 1916, while serving as an infantryman ; after he had recovered from his wounds he was given a commission in the Royal Regiment of Artillery, and he saw active service again in Flanders with a Heavy Battery until the armistice. Calthrop's war service was of a pattern with his whole life : he was the last man one would expect to see sitting in a job at the Base, or collecting decorations on lines of communication.

After the war he entered at Pembroke College, Cambridge, taking a Natural Science Tripos, and in 1921 he joined the Physics staff of Queen Mary (then East London) College, under Professor C. H. Lees. For a period after his appointment he did research work in general physics and heat, publishing a number of papers in the *Philosophical Magazine* and in our *Proceedings*, but he soon decided that teaching was his real vocation, and for the last twenty years of his life he devoted himself almost entirely to the training of students and to doing all he could for their general welfare. He had a real affection for the young, and he never spared himself in their service. His interest in teaching is shown in a number of notes he published in the *School Science Review*, and in a highly original course of *Advanced Experiments in Practical Physics* which appeared in 1938 and was very favourably reviewed both here and abroad. His last publication, at the end of 1943, was a *German Physics Reader*.

His chief handicap as a teacher was an unconquerable diffidence of manner. He built up a fine library of modern physics books in English, French and German, which he studied assiduously (and, to mention his only vice, under-scored freely), he took pains to find the best way of treating a subject and he prepared admirable lectures—which he then, in class, delivered in a low tone and an almost apologetic manner, as though he were ashamed of their content. As a result, although the best students got a lot from him, the less discerning tended to under-value his teaching, and I fear that he was often conscious of this. His most effective teaching was done in the laboratory and in informal talks with the students, to whom he willingly sacrificed much of his spare time.

Calthrop was one of the most transparently honest and least self-seeking of men. Although of a naturally retiring disposition, and a bachelor, he was—as I have known since his earliest contacts with my own family—entirely at his ease with very young children, and outstandingly happy in his relations with them.

From 1939 to 1945, while Queen Mary College was evacuated to Cambridge, Calthrop resided in King's College. He enjoyed immensely the privilege of sharing in the corporate life of a great Cambridge College, and his obvious integrity, his eagerness to help and his evident delight in his surroundings brought him many new friends. There is much comfort in the thought that his last years, spent in such congenial surroundings, were among the happiest of his life.

H. R. ROBINSON.

JOHN JOB MANLEY

Manley was born on 13 July 1863 near Wellington, Somerset, and the family migrated later to Devon. At the age of 25 he became Daubeny Curator of the Laboratory of Magdalen College, Oxford, and after the first three years he taught practical physics and analytical chemistry to undergraduates reading for the Honours School of Natural Science. His first research work at this laboratory was carried out in collaboration with V. H. Veley

and resulted in a series of papers on the preparation of anhydrous nitric acid, and on the physical properties of aqueous solutions of this acid and of sulphuric acid. This work, which was completed in 1905, was recognized by the award of an M.A. degree, *honoris causa*, in 1903.

Manley was typical of the best school of experimentalists—those who took a pride in carrying out operations with their own hands. His opportunities for research were increased by the award of a Research Fellowship in 1917, the year in which he took his M.A. He gave up his teaching work in 1923 and his curatorship in 1929, in which year, at the age of 66, he left Oxford and equipped a laboratory in Bournemouth. He took an Oxford D.Sc. in 1931. He continued work at Bournemouth until 1938 as the holder of a Research Fellowship and then privately until his death on 24 March 1946, which occurred after only a fortnight's illness.

After his partnership with Veley had ceased, Manley devised a differential densimeter for the study of sea-water ; a duplicate of this apparatus, mounted on gimbals, was successfully used by N. P. Campbell on a voyage from London to Colombo. Later, whilst standardizing a set of weights, Manley encountered certain difficulties which led him to devise his enclosed or " protected " balance beam (1910), and ultimately to the revision of Landolt's work on the apparent change in weight during chemical reaction (1912). For this revision he chose the straightforward reaction between barium chloride and sodium sulphate, instead of the somewhat complicated ones selected by Landolt. He found that the change in weight did not exceed one part in 100 million. His interest in the subject of the precision balance and the study of analytical weights continued up to his last illness, and several papers were published in the *Proceedings of the Physical Society*, the last of them, on the advantages of rhodium plating, only last year (*Proc. Phys Soc.* **57,** 136 (1945)).

His experimental skill, and in particular his mastery of the art of glass blowing, was successfully applied to the study of extremely thin wires for platinum thermometers, gases at low pressures, and the interaction between mercury and helium. His method of purifying phosphoric acid by ozonized air, worked out in 1922, was used to good effect in 1929 to 1934 in an extension of Baker's work on the intensive drying of benzene.

J. J. Manley's chief recreations were those of a quiet, modest man, who liked the open air—cycling, climbing, gardening and photography. He was a devout churchman, with an assured faith in things spiritual. Many will remember him as an unassuming worker, an interesting speaker, with that self-confidence which comes from certainty of knowing his subject. He was twice married, and is survived by his second wife and by his son, C. H. Manley, the City Analyst of Leeds and Wakefield. One of his two grandsons is a civil engineer.

RAOUL FREDERIC SCHMID

R. F. Schmid, Fellow of the Physical Society, died in Budapest on 2 October 1943. He was carried away by a disease of tragic suddenness at the age of 39. A man of science, he worked and taught all his life. As a true scientist he professed the freedom of thought and human rights in an age when this was dangerous and difficult, trusting a better period would come. This period, however, he did not live to see. With a loving heart he helped everybody, teaching also through the example of his own life.

At the age of 22 he had graduated at three Universities and possessed the diplomas of chemical and economic engineer and the degree of Ph.D. He was the winner of several scholarships and, as a Rockefeller Foundation Fellow, worked in Chicago in 1932. Originally a chemist, he saw in molecular spectra the firm experimental ground for studying the fundamental problems of chemistry. Working in this field, he added something in nearly every domain of molecular spectroscopy with an admirable fertility. He united wide knowledge and exceptional memory with experimental skill and invention.

Through the inscrutable ways of providence, his collaborator, L. Gerö, was destined to follow him in the short space of one year and a half. He died, at the age of 35, a victim of the war, at a moment when he could already expect to have got to the beginning of the eagerly expected new period. Their names are now closely connected by the scientific work they have done together.

To name only some milestones of this work: a minute investigation of the CO spectrum in order to get as complete an analysis as possible of the term system of this molecule, of such importance from the point of view of organic chemistry; pioneer work in discovery and interpretation of perturbations and predissociations; establishment of the dissociation schemes for CO, CN, N_2, NO and diatomic hydride and deuteride molecules; an interpretation of the thermochemical data of organic chemistry and the emphasizing of the importance of excited atomic states from the point of view of chemistry.

Those who part from this world continue to live in their works and in the hearts of those who knew and loved them. J. G. VALATIN.

REVIEWS OF BOOKS

Mathematical Theory of Plasticity, by W. PRAGER. (Brown University, Providence, Rhode Island, 1944.)

A considerable literature now exists dealing with various aspects of the mechanics of solids beyond the elastic range. Only in one or two instances, however, as far as the writer is aware, has any attempt been made to co-ordinate the available information in the form of treatises dealing with the subject as a whole, and none of these treatises appears to merit the description of " mathematical theory of plasticity ".

The present manuscript therefore meets a considerable need, giving as it does a summary of the present state of knowledge of mechanics of solids beyond the elastic range, together with a study of the mathematical theory of plasticity as applied to certain types of ideal materials.

In the opening two chapters the analysis of stress and strain is developed in terms of both Lagrangian and Eulerian co-ordinates, and some properties of the stress and strain tensors are discussed, particularly in relation to certain invariants of the stress and strain systems. Expressions for " work " are obtained in terms of the stress and strain tensors. These analyses are the basis of the consideration of stress-strain relations in subsequent chapters.

The third chapter considers the relations between the stress and strain tensors for an isotropic elastic body. For this purpose the stress and strain tensors are decomposed into functions of the mean stress or strain and the stress and strain deviators. The equations of equilibrium and compatibility are derived, and the relations between the invariants of stress and strain are considered. An expression for strain energy is obtained.

The nature of elastic breakdown is discussed, with consideration of the various theoretical criteria, and experimental evidence supporting them.

In Chapter IV the stress-strain relations for a visco-plastic material (defined as a viscous fluid having a definite yield point) are derived from consideration of the corresponding relations for a viscous fluid, by the use of the tensor of " over stress " expressing the extent by which the yield limit has been exceeded. The relations corresponding to several arbitrary conditions of yielding are stated.

Experimental methods of determining the coefficient of viscosity of a visco-plastic material are described. As a limiting case of the visco-plastic material, the stress-strain relations of a perfectly plastic material are derived, and the experimental evidence of Taylor and Quinney in this connection is discussed.

Chapter V deals with the effects of strain hardening on the stress-strain relations of a visco-plastic or plastic body, suitable modification being made of invariants of stress and strain concerned in the relations to account for this effect. Both isotropic and anisotropic strain hardening are considered. The latter form is assumed to be such that the three principal planes of strain constitute three equivalent planes of asymmetry, and that rotations interchanging principal axes of strain do not change the value of invariants.

In Chapter VI, the stress-strain relations of composite material are derived from the relationship obtained in previous chapters for the relatively simple materials. Materials composed of two or more of the following elements are considered—elastic solids, viscous fluids, visco-plastic materials and perfectly plastic materials. These materials include

those of Voigt and Maxwell. The constituent materials are either placed in series, with superposition of deformations and the same stress; or in parallel, with superposition of stresses, and the same deformation. The modification of the stress-strain relations for a composite visco-plastic and elastic material used by F. Odquvist for the analysis of creep is stated.

Chapter VII deals with the subject of materials displaying a gradual transition from the elastic to the plastic state. Geometrical methods of representation of stress and strain deviations are first evolved. Using these methods of analysis, and making the assumption that materials of this type obey a general relation of the type

$$dq = dp\left\{\frac{1-(q_1{}^2)^n}{C^2}\right\},$$

where q is stress and p is strain, it is shown that the equation relating the stress (S_1k) and strain $(\epsilon_1 k)$ tensors is of the type

$$A\frac{\partial}{\partial t}\frac{(S_1k)}{A} = 2\mu\,\dot{\epsilon}_1 k, \qquad \text{where} \qquad A^{2n} = 1 - \frac{(S_{11})^{2n}}{C}.$$

C, μ and n are constants, t is time and S_{11} a second-order invariant. Modification of the stress and strain relations for materials of this type undergoing isotropic or non-isotropic strain hardening is discussed.

In Chapters VIII to XI the author shows the application of the theory of plasticity to the problems of pure shear, torsion of a cylindrical body, and plane strain with and without axial symmetry. In each case general formulae are first developed, and are subsequently applied to the cases of the elastic, visco-plastic, St. Venant and Prandtl-Reuss materials, and also to materials with gradual transition from the elastic to the plastic state. Where precise mathamatical solutions of the problems presented are not found possible, reference is made to various experimental methods of obtaining the values of stress or strain functions, including the flow, electric field, and membrane analogies. Numerous references are given to the publications of other workers who have dealt with these particular problems.

In the concluding chapter two boundary-value problems involving the variation of the stresses and displacements in certain areas of incompressible bodies, including elastic material, Prandtl-Reuss material and material with gradual transition to the plastic state, are considered. Solutions are obtained by the employment of variational principles postulating minimum values for certain combined functions of stresses and displacements.

Two minor criticisms have occurred to the reviewer. Regarding the content of the manuscript, it seems a matter of some regret that in view of the practical importance of the subject of creep or time-dependent strain, the author has given only passing notice to it.

Regarding the actual arrangement of the text, the provision of a glossary of symbols used would have improved the readability of the manuscript, since these symbols are quite numerous and constitute a considerable tax on the reader's memory.

It is to be hoped that in a future manuscript the author may apply the theory developed in this present work to further problems of plasticity, many of which, e.g. those involved in the engineering design of gas turbines, pipe flanges, and aerofoils, are now acquiring very considerable practical importance.

A. E. JOHNSON.

Mathematical Theory of Optics, by R. K. LUNEBERG. Pp. xvii–401. (Brown University, Providence, Rhode Island, 1944.) $4.

These mimeographed notes are a record of a course of lectures given at Brown University in the summer of 1944. The subject-matter is dealt with in a comprehensive and unbiased manner, and the book should prove of value to those looking for a thorough treatment of the theory underlying image formation in, and the design of, optical systems. It is likely to be of interest to those designers who make use of trigonometric ray tracing and are seeking an alternative method. No mention is made of such tracing, the theory being developed from a point of view which differs fundamentally from, say, that adopted in the well known book by Conrady. Considerable use is made of the characteristic

function method and the power and breadth of application of this method is amply demonstrated. The book may be regarded as a companion to the Cambridge Tracts by Synge and by Steward and the classic, *Optik*, by Professor Max Born.

In the first chapter the wave theory (based on Maxwell's equations) and the ray theory of optics are developed simultaneously. Transparent isotropic media are considered. The kernel of this theory is the equation of the wave fronts (the eiconal equation),

$$\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial z}\right)^2=n^2(x,y,z),$$

the light rays being the orthogonal trajectories of a set of wave fronts, $\psi=ct$. Differential equations satisfied by the light rays are found and Fermat's principle is introduced. It is shown that wave fronts may be constructed with the aid of light rays ; and, conversely, by means of Jacobi's theorem, a method is obtained for constructing the light rays when a solution to the above equation is known. Applications are made to (*a*) spherical waves in a homogeneous medium, (*b*) wave fronts in a medium of discontinuous optical properties, (*c*) periodic waves of small wave-length.

Hamilton's theory is presented in Chapter II. The three characteristic functions, the point-, mixed-, and angle-functions, are defined and their properties and physical significance are studied. Integral invariants and their application to ray congruences are discussed and an interesting geometrical interpretation is given of Poincaré's integral invariant. There are a few examples of the characteristic functions, but the important applications are reserved for Chapter III, in which some of the topics studied are the correction of optical instruments by aspheric surfaces, axially symmetric systems, spherical aberration and coma, and the problem of designing condenser systems.

A comprehensive treatment of first-order optics is given in Chapter IV. Considerable use is made of a matrix notation in the manipulation of the mapping equations. The problem of first-order optics is to investigate the properties of the four-parameter system, S, of rays which lie in the neighbourhood of a given ray, for example the principal ray. This is shown to be equivalent to a study of the linear transformations

$$\begin{pmatrix}\boldsymbol{X}_1\\ \boldsymbol{P}_1\end{pmatrix}=\begin{pmatrix}\boldsymbol{A}\,\boldsymbol{B}\\ \boldsymbol{C}\,\boldsymbol{D}\end{pmatrix}\begin{pmatrix}\boldsymbol{X}_0\\ \boldsymbol{P}_0\end{pmatrix},$$

where $\boldsymbol{X}_i=\begin{pmatrix}x_i\\ y_i\end{pmatrix}$, $\boldsymbol{P}_i=\begin{pmatrix}p_i\\ q_i\end{pmatrix}$ $(i=0,1)$ and the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$, each of order two, are such that

$$\boldsymbol{A}^*\boldsymbol{C}-\boldsymbol{C}^*\boldsymbol{A}=0,\quad \boldsymbol{B}^*\boldsymbol{D}-\boldsymbol{D}^*\boldsymbol{B}=0,\quad \boldsymbol{D}^*\boldsymbol{A}-\boldsymbol{B}^*\boldsymbol{C}=\boldsymbol{1}.$$

For this purpose Hamilton's method of characteristic functions is used, and the results are interpreted optically in regard to the formation of images by the narrow bundle, S. When $|\boldsymbol{C}|\neq 0$, it is possible, using the above matrix equation, to express $\boldsymbol{X}_0$ and $\boldsymbol{X}_1$, in terms of $\boldsymbol{P}_0$ and $\boldsymbol{P}_1$, in the form

$$\begin{pmatrix}\boldsymbol{X}_0\\ -\boldsymbol{X}_1\end{pmatrix}=\begin{pmatrix}\tilde{\boldsymbol{A}}+\gamma_0\boldsymbol{1}, & -\boldsymbol{F}\\ -\boldsymbol{F}, & \tilde{\boldsymbol{C}}+\gamma_1\boldsymbol{1}\end{pmatrix}\begin{pmatrix}\boldsymbol{P}_0\\ \boldsymbol{P}_1\end{pmatrix},$$

where γ_0 and γ_1 are constants and $\tilde{\boldsymbol{A}}$ and $\tilde{\boldsymbol{C}}$ are symmetric. In the case of an axially symmetric system we obtain the Gaussian first-order optics. Then

$$\tilde{\boldsymbol{A}}=\begin{pmatrix}a & 0\\ 0 & a\end{pmatrix},\quad \boldsymbol{F}=\begin{pmatrix}f & 0\\ 0 & f\end{pmatrix},\quad \tilde{\boldsymbol{C}}=\begin{pmatrix}c & 0\\ 0 & c\end{pmatrix},$$

and by means of these matrices the properties of the Gaussian system are deduced. A generalization is made to the case of a bundle of rays about an oblique meridional ray in a system of revolution. Then

$$\tilde{\boldsymbol{A}}=\begin{pmatrix}a_1 & 0\\ 0 & a_2\end{pmatrix},\quad \boldsymbol{F}=\begin{pmatrix}f_1 & 0\\ 0 & f_2\end{pmatrix},\quad \tilde{\boldsymbol{C}}=\begin{pmatrix}c_1 & 0\\ 0 & c_2\end{pmatrix},$$

and we have what is called an orthogonal ray system.

The theory of the third-order Seidel aberrations in a system of axial symmetry is given in Chapter V. The image functions are developed one stage beyond the first order in the neighbourhood of the principal ray. The usual expressions are found for the five image errors, and, in the case of a medium with a continuously variable refractive index, the analogous integral expressions are given. Petzval's theorem on field curvature is discussed and a section deals with chromatic aberrations.

The last chapter deals with the Diffraction Theory of Optical Instruments, that is, the propagation of light waves in a non-homogeneous optical medium. Here we return to the electromagnetic theory. The diffraction of unpolarized light and the diffraction patterns for various types of aberrations are discussed. Problems of resolution also receive attention, including the resolution of objects of periodic structure. Some valuable supplementary notes are given. About forty pages are devoted to the theory of electron optics, and there are some notes by Hertzberger, including information on the optical qualities of glass.

After this survey of the contents of the book little need be said further. It is a pleasure to find that the characteristic function method is emphasized and used extensively. More publicity on behalf of this method might be given if the notes were used as the basis for a printed book more readily available than the mimeographed notes from Brown University.

L. S. GODDARD.

Table of Arc sin x. Pp. xx + 124. \$3.50 ; *Tables of Associated Legendre Functions.* Pp. xlvi + 306. \$5.00. (Columbia University Press, New York: British agents, Scientific Computing Service, Ltd., 23 Bedford Square, London, W.C.1, 1945.)

These volumes are numbered (17) and (18) in the, by now, well-known series of tables produced by the New York Mathematical Tables Project, directed by Dr. Arnold N. Lowan, and now sponsored by the National Bureau of Standards. They are typical and worthy members of the series.

The purpose of the first volume is to enable 12-decimal values of arc sin x in radian measure to be obtained for any value of x from zero to unity. The function is, of course,

$$\int_0^x \frac{dt}{\sqrt{(1-t^2)}},$$

and since previous volumes have tabulated the transcendental functions (natural logarithm and inverse tangent) arising in the integration of rational algebraic functions, the present table may be regarded as the first instalment of tables of integrals of functions involving the square root of a quadratic. It is believed that work has also been done on the companion functions, arc sinh x and arc cosh x, and it is to be hoped that the publication of similar tables of these functions may be not long delayed.

The main Table gives 12-decimal values of arc sin x at interval 0·0001 from $x=0$ to $x=0{\cdot}9890$. In this range the greatest attainable accuracy in interpolation is obtained by the use of the second differences, or " modified " second differences, provided. The difficult range, for greater values of x, has been treated as follows. First, the interval has been reduced to 0·00001. Second differences suffice to $x=0{\cdot}99750$, and modified second differences thence to $x=0{\cdot}99900$. Fourth differences are given from 0·99900 to 0·99950, but no differences in the remaining range. Here interpolation is to be effected by use of the formula

$$\text{arc sin } (1-v)=\sqrt{(2v)}.f(v),$$

and $f(v)$ is tabulated at interval 0·00001 from $v=0$ to $v=0{\cdot}00050$. Tables of interpolation coefficients for use with the differences provided are included and worked examples of their use are given in the introduction.

The introduction gives also a full account of the methods of computation and checking adopted and a bibliography. It is clear from the latter that the present Table has more decimals and/or a smaller interval than any of its predecessors, and that it constitutes a fundamental table of great value.

Considering the importance of the Legendre and Associated Legendre functions (e.g., in spherical, ellipsoidal, and toroidal harmonic analysis), it is somewhat surprising that tables of these functions have not been more readily available. The present volume goes far to fill the gap. It tabulates the functions $P_n^m(x)$ and $Q_n^m(x)$ and their first derivatives for integer values of n from 0 to 10 and of m from 0 to 4 (or to n if $n<4$), and for half-odd-integer values of n from $-1/2$ to $9/2$ and integer values of m from 0 to 4, and for real and pure imaginary values of x. The argument ranges from 1 (in some cases 0) to 10 at interval 0·1. Values of $P_n^m(\cos\theta)$ and $d\{P_n^m(\cos\theta)\}/d\theta$ for θ from 0° to 90° at interval 1° are also given. Normally six significant figures are tabulated. No provision for interpolation is made, but a schedule showing the accuracy attainable by various Lagrangian interpolation formulae precedes each main Table. Some supplementary tables give exact values and normalization factors.

It is clear that this volume will be of very great use to physicists and others whose work requires numerical values of the functions tabulated in it, and the accuracy should be ample for almost all practical applications. W. G. BICKLEY.

Thermodynamics, by PHILIPP FRANK. Pp. iv + 123 + 24. (Brown University, Providence, Rhode Island, 1945.)

These notes, in mimeographed form, were prepared in connection with one of the very valuable summer schools run at Brown University. The scope of the course is, roughly speaking, that covered by the ordinary textbook, restricted to the classical (and perfectly general) thermodynamics ; the statistical approach is not employed. The course, however, is far from being conventional. It is original in being one of the first to adopt Caratheodory's methods, so that, for example, the reader sees clearly that *entropy* is a concept suggested by, and associated with, the first law of thermodynamics, and not, as students commonly suppose, only introduced because of the second law. This fact was, as it happens, known to Kelvin, but it has tended to slip out of sight. In any case, Caratheodory's treatment is of an altogether higher order of rigour than that of any of his predecessors. This is not to say that it uses more difficult mathematics, but its outlook is more critical.

After the bases of the science have been laid, the notes proceed to deal with the applications to pure substances, engines, magnetic and electric phenomena ; the third law is then introduced and its application to problems of chemical equilibrium described. A valuable note on the thermodynamics of gas mixtures is contributed by John A. Goff. In this, the methods of Gibbs are supplemented by recent developments, including the determination of free energies (or equivalent quantities) from spectroscopic data.

Whilst we may envy those who were privileged to attend the summer school, and may find these notes useful and interesting, it would not be true to say that they would make a good book if published as they stand. For that, they would need either to be pruned to make more definitely a set of notes, or else expanded further to give those full explanations which were doubtless added orally when the lectures were delivered.

J. H. A.

Atomic Artillery and the Atomic Bomb. By J. K. ROBERTSON. Pp. viii + 173. (New York: Van Nostrand Co., 1945.) $2.50

The title of this book appears more sensational than the author really intended, for his artillery is, in fact, alpha particles, protons, electrons and the like. The book is intended for the intelligent layman, and covers roughly the work on atomic physics subsequent to the discovery of the electron by Sir J. J. Thomson. After an explanation of the nature of elements and the significance of atomic weights, the author discusses the concept of the atom, proceeds to the electron and discusses methods of accelerating charged particles. He then mentions the early work on positive rays and the mass spectrograph, gives a short account of radio-activity, and then comes to gamma rays. After this, he deals with the work on cosmic rays which, in the period 1930–35, gave us the new particles of positron and meson (or, as he calls it, the mesotron).

The work with which Rutherford's last period of activity was associated was the transmutation of elements by bombardment with various particles, and the author gives a clear account of the nature of this nuclear chemistry, explaining how both mass number and atomic weight must balance in each reaction. From this point his story becomes quite modern, and we are introduced to fission and chain reactions, and it is explained how these could be used to keep a reaction going, and thus provide the world with a new source of power.

As is now well known, this work led not only to a possible source of power, but incidentally to a means of production of elements not previously known and, in particular, of plutonium. Some short account of this work is given and its application to the atomic bomb is mentioned.

The book, on the whole, seems accurate. There is one slip where *positron* occurs for *proton*, and the reader is led to suppose that Aston is still alive and working, but these seem to be the major slips.

It is satisfactory to read a book on this subject which does not spin words at great length on the sociological and political implications of the work being done, and it is still more satisfactory to know that we can now recommend to our non-scientific friends a book which they can read for themselves and understand in the knowledge that they will not be misled as to the facts.

J. H. A.

Circular and Hyperbolic Functions, Exponential and Sine and Cosine Integrals, Factorial Function and Allied Functions, Hermitian Probability Functions. (*British Association Mathematical Tables, Volume I*). Second edition. Pp. xi+72. (London: Cambridge University Press, 1946.) 10*s*.

The collection under review is a welcome second edition of the British Association Mathematical Tables, Volume I, originally published in 1931 and now for some years out of print. The functions tabulated (together with even differences, sometimes up to the sixth) are as follows: $\frac{1}{2}\pi x$[1(1)100 ; 15], $\sin x$, $\cos x$ [0·0(0·1)50·0 ; 15 and 0·000 (0·001)1·600 ; 11], $\sinh \pi x$, $\cosh \pi x$ [0·0000(0·0001)0·0100 ; 15 and 0·00(0·01)4·00 ; 15], $\sinh x$, $\cosh x$ [0·0(0·1)10·0 ; 15], $\mathrm{Ei}(x)$, $-\mathrm{Ei}(-x)$ [0·0(0·1)15·0 ; 11], $\mathrm{Si}(x)$, $\mathrm{Ci}(x)$ [0·0(0·1)20·0(0·2)40·0 ; 11, 10], $x\,!$ [0·00(0·01)1·00 ; 12], $\left\{1+\int_0^x \log_{10} t\,!\; dt\right\}$ [0·00(0·01) 1·00 ; 10], $\frac{d^n}{dx^n}\log_e x\,!$ [n=1, 2, 3, 4, 0·00(0·01)1·00 and 10·0(0·1)60·0 ; 10, 12], $Hh_n(x) = \int_x^\infty Hh_{n-1}(x)\,dx$ where $Hh_0(x) = \int_x^\infty e^{-\frac{1}{2}t^2}\,dt$ [n=0(1)21, x=−7·0(0·1)+6·6 ; 10], $Hh_0(x)$ $Hh_2(x)/\{Hh_1(x)\}^2$ [−7·0(0·1)+50·0 ; 9 to 3]. In tabular content the new edition thus does not differ from the old, but the opportunity has been taken to provide an auxiliary table extending the values of the Factorial Function, $x\,!$, from 12 to 18 places of decimals and to correct eleven errors (of which nine merely affected end-figures by one unit) that have been detected. Possessors of the first edition are reminded that they may obtain a list of these corrections by applying to the Office of the British Association at Burlington House, London W. 1.

A drastic pruning of the new edition of the tables has removed the historical note, and the 30-page introduction which formed such an interesting and informative a feature of the first edition. This loss may not be felt acutely by those who, as it were, already possess a function and merely require its numerical values, but, from the standpoint of general interest and suggestiveness for fruitful application, one experiences a feeling of slight regret at the omission of the material illustrating the analytical properties and uses of the " Polygamma " Functions and the repeated integrals of the Error Function (now called Hermitian Probability Functions).

Mathematical tables, although (when accurate) essentially perdurable, are yet susceptible to loss of pre-eminence by comparison with subsequent more extensive tables. Few users will find the number of significant figures inadequate in the present volume of British Association tables, but, as is obvious from the list of contents given above, for many

purposes subtabulation of several of the functions would be welcome, although inevitably bulky. In this connection it is relevant to remark that since 1931 a new standard for the close tabulation of a large number of functions has been established by the publications of the New York Mathematical Tables Project.

Criticism of a book because it is not a whole library is, however, unfair. The volume under review contains a diverse, interesting and most useful collection of tables which yield their full value to those prepared to interpolate accurately. The price is very moderate, despite the fact that the present binding is not sufficiently stout for its hard future. It is a pleasure to consult tables so elegantly set and printed. M. S. JONES.

The Music Review, Vol. VII, No. 1. (Heffer & Sons, Cambridge.) 5*s*.

This number contains an article by B. van der Pol on " Music and Elementary Theory of Numbers ", which deals with time and rhythm, absolute and relative pitch, Euler's theory of dissonance, musical scales, relaxation oscillations and variations of musical pitch. The author's investigations on the variation in pitch between orchestras which broadcast was described in Vol. VII of the *Progress Reports* (pp. 33–34). The remaining topics are considered either in Helmholtz's *Sensations of Tone* or in modern textbooks of sound, but the musical physicist will find in this article a useful summary of present knowledge in this field, and also some useful observations on the sense of absolute pitch, studied by the author, himself a musician with this gift and with the scientist's sense of impartiality. E. G. R.

Statistical Thermodynamics, by ERWIN SCHRÖDINGER. Pp. 88. (Cambridge: The University Press, 1946). 6*s*.

This very small book contains a great deal of material, which was originally presented as a course of lectures at the Dublin Institute for Advanced Studies, in the spring of 1944. It is hardly a reference work to which a man would turn when presented with a specific problem which he wished to solve, but rather a background exposition which he should read and ponder when he is not busy applying statistical methods to particular problems.

Schrödinger adopts the attitude that there is only one problem in statistical thermodynamics, that of determining the distribution of an amount E of energy over N similar systems, which may be only mental copies of one real system. From this starting point he considers how the distributions may be enumerated, and in doing so he is led to discuss the logical bases of the procedure, and also to sketch the mathematical apparatus normally used. He is more interested in the difficulties of the theory than in those parts where all is straightforward, and consequently has written a most stimulating book, which is well worth its price. Incidentally, though produced under war-time conditions, the book is pleasant to handle and to look at—a tribute to the skill of the printers. Among matters which are discussed are the setting equal to zero of the constant entropy which remains in a crystalline system at absolute zero, and the relationship between the classical, the Bose-Einstein and the Fermi-Dirac statistics. All these are subjects on which a great deal has been written before, but the author's peculiar quality of illuminating those matters on which he writes makes his treatment very well worthy of study. J. H. A.

CORRIGENDUM

" Crystal structure of double oxides of the perovskite type ", by HELEN D. MEGAW (*Proc. Phys. Soc.*, **58**, p. 133).

The address given under the author's name should read " Philips' Lamps, Limited, Mitcham, Surrey ; now at Birkbeck College, London ".

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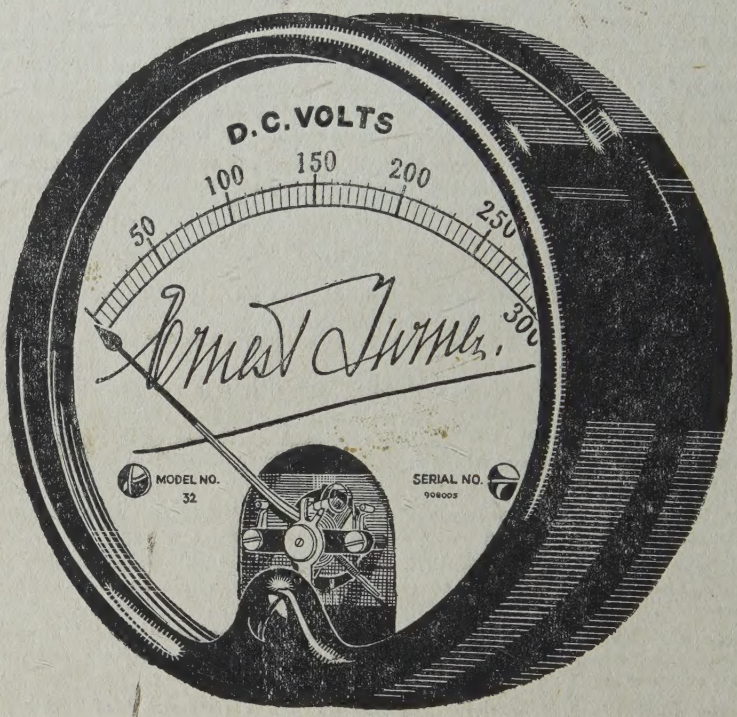

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